THE MAGIC OF
NUMBERS

by

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Note

The topic of this book is described in the first few pages of the opening chapter. It concerns what may be the least expected turn of scientific thought in twenty-five centuries. Should this return to a remote past—for that is what the most recent philosophy of science really is—be generally accepted, our descendants a few generations hence will look back on us and our science as incredibly unenlightened.

Not much of the proposed substitute for the scientific method as commonly understood has been discussed outside professional scientific circles. An untechnical account of the origins and progress of the new approach to nature may therefore be of interest to those who do not make their livings at science. It will appear that the new and the old are strangely alike.

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Contents

The Past Returns

THE hero of our story is Pythagoras. Born to immortality five hundred years before the Christian era began, this titanic spirit overshadows western civilization. In some respects he is more vividly alive today than he was in his mortal prime twenty-five centuries ago, when he deflected the momentum of prescientific history toward our own unimagined scientific and technological culture.

Mystic, philosopher, experimental physicist, and mathematician of the first rank, Pythagoras dominated the thought of his age and foreshadowed the scientific mysticisms of our own. So varied was his genius that the crassest superstitions and the most uncompromising rationalisms might appeal to his authority—"Himself said it"—all down the Middle Ages. The essence of his teaching was the mystic doctrine that "Everything is number." With Galileo's revival in the late sixteenth century of the experimental method in the physical sciences, a method in which Pythagoras had pioneered nearly twenty-two centuries earlier, number mysticism passed out of science.

The seventeenth century saw the creation by Newton and Leibniz of a new mathematics, devised to bring the continu-

ously varying flux of nature under the domination of rigorous reasoning. Combining this dynamic mathematics with precise observation and purposeful experiment, Newton and his followers in the seventeenth and eighteenth centuries fixed the modern scientific method for the astronomical and physical sciences. The form they gave it was to stand unchallenged till the third decade of the twentieth century.

The aim of this method was twofold: to sum up the observable phenomena of the physical universe in readily apprehended generalizations—called by their inventors or discoverers "laws of nature"; to enable human beings in some degree to predict the course of nature. Always observation and experiment were the first and last court of appeal. However reasonable or however inevitable the verdict of mathematics or other strict deductive reasoning might appear, it alone was not accepted without confirmation by this final authority.

The successes of the method heavily overbalanced its failures all through the nineteenth century and well into the twentieth. In less than two centuries the application of scientific technology to industry wrought a profounder transformation of western civilization than had all the wars and social upheavals of the preceding four thousand years.

Concurrently with this vast revolution in the material world, equally subversive changes from time to time overthrew established creeds that had possessed the thoughts of men for scores of generations. The universe disclosed by science was not always that of revelation and tradition, nor even that which a supposedly infallible logic insisted must be the fact. Here also the absolutes of more than two thousand years were impartially scrutinized. Those that had proved

barren of positive knowledge were ruthlessly abandoned. The unaided reason as an implement of discovery and understanding in the exploration of the material universe dropped out of use. Its sterility in science then cast suspicion on it in its own traditional domain. Of what human value were truths immune to any objective test that human beings might invent? Protests that truths other than those of science exist timelessly in a realm of Eternal Being, and are forever inaccessible to the finite reach of science, were silenced by the dictum, "Experiment answers all." Then, quite suddenly, about the year 1920, the most positive of all the sciences began to hesitate.

By the middle 1930's a few prominent and respected physicists and astrophysicists had reversed their position squarely. Facing the past unafraid, they strode boldly back to the sixth century B.C. to join their master. Though the words with which they greeted him were more sophisticated than any that Pythagoras might have uttered, they were still in his ancient tongue. The meaning implicit in their refined symbolisms and intricate metaphors had not changed in twenty-five centuries: "Everything is number." He understood what they were saying.

The retreat from experiment to reason was applauded by some philosophers and scientists, deplored by others. But the fact in the new movement was beyond dispute. Either the leaders had gone back to Pythagoras to acknowledge that he had been right all these centuries, or he had come forward to convince them that the modern scientific method of Galileo and Newton is a delusion.

On a first, exploratory pilgrimage to the past the daring ultramoderns had lingered for a few moments in the shadow

of Plato. Quickly realizing that in all matters pertaining to the mysteries of number here was only the pupil, they sought his master. Two centuries before Plato was born, Pythagoras had believed and taught that the pure reason alone can reveal the truth of anything; observation and experiment are snares to trap and betray the unwary senses. And of all languages in which constant knowledge as opposed to variable opinion may be described, that of number is the only one on which the pure reason may safely rely. "Himself said it," and now, twenty-five centuries after his historic death, he was repeating himself in the language of a nascent science.

A devout believer in the doctrines of reincarnation and the transmigration of souls, Pythagoras may at last have found a congenial habitation in the sheer abstractions of twentieth century theoretical physics. "For my own lapses from the one true faith," he may now reflect, "I was condemned to spend life after life in the vile dogmas of false philosophers and in the viler imaginings of base numerologists. But now I am unbound from the Wheel of Birth. When I experimented with my hands and my hearing to discover the law of musical intervals, I sinned against the eternal spirit of truth, defiling my soul with the unclean things of the senses. Then I beheld the vision of Number, and knew that I had betrayed my better part. By proclaiming the truth that everything is number, I sought to cleanse my soul and gain release from the Wheel. But it was not enough. Few believed and many misunderstood. To expiate my transgression I passed through that purgatory of error and falsehood, a name honored in the mouths of fools. Now I discern an end to my torment in the dawn of a new enlightenment which was already old ages before I was Pythagoras. The deceptions of

the senses shall mislead mankind no more. Observation and experiment, the deceitful panders of sensory experience, shall pass from human memory and only the pure reason remain. Everything is number."

The master's prophecy becomes less abstract and closer to the scientific actualities of the twentieth century. Speaking as a mathematical physicist and mathematical astrophysicist he proceeds to details. "I believe . . . that all the laws of nature that are usually classed as fundamental can be foreseen wholly from epistemological considerations." In a brief aside he reminds us that epistemology is that department of metaphysics which deals with the theory of human knowledge. To preclude any possible misunderstanding of his meaning he elaborates his heretical creed. "An intelligence unacquainted with our universe, but acquainted with the system of thought by which the human mind interprets to itself the content of its sensory experience, should be able to attain all the knowledge of physics that we have attained by experiment. He would not deduce the particular events and objects of our experience, but he would deduce the generalizations we have based on them. For example, he would infer the existence and properties of sodium, but not the dimensions of the earth."

If Pythagoras—ventriloquizing thus in 1935 through Sir Arthur Eddington, a leader in the retreat to the past—should be right, it would seem that the experimental scientists since Galileo and Newton have gone to much unnecessary labor to discover the obvious and proclaim it in truisms. If it is false that experiment answers all, it may be true, as some of the ancients believed, that reason answers all, or, as the successors of Pythagoras seem to believe, nearly all. For, as we

have just been cautioned, reason may be unable to deduce the diameter of the earth from any data wholly within the human mind. But this defect is entirely negligible in comparison with the ability to foresee the existence and properties of the chemical elements "wholly from epistemological considerations."

By taking sufficient thought the scientific epistemologist may rediscover for himself, without once rising from his chair in an otherwise empty room, all that three centuries of observation and experiment since Galileo and Newton have taught us of the "fundamental laws" of mechanics, heat, light, sound, electricity and magnetism, electronics, the constitution of matter, chemical reactions, the motions of the heavenly bodies, and the distribution in space of stellar systems. And by the same purely abstract considerations the thoughtful epistemologist may attain verifiable knewledge of natural phenomena which are still obscure to science, for example, the internal motions of the spiral nebulae.

Should only some of these impressive claims be sustained, the twentieth century return to Pythagoreanism may be remembered ten thousand years hence as the dawn of a lasting enlightenment and the end of the long night of error which descended on western civilization in the seventeenth century. The costly apparatus of our laboratories and observatories will have crumbled and rusted away, except possibly for a few relics fearfully preserved in the World Museum of Human Error. Above the entrance the guardians of public sanity will have inscribed the truths that liberated mankind: "Experiment answers nothing. Reason answers all." To balance these, the same guardians will have embellished the pediment of the Temple of Knowledge and Wisdom with the summary of

the universe and a solemn warning: "Everything is Number. Let no man ignorant of Arithmetic enter my doors."

But all this is in the calm certainty of a golden age of the future while we, unhappily, must endure the steel and errors of the present. To mitigate our lot we may return to the past for an hour or two, to read in it the certainty of our present and the hope of our future.

What shall we ask the past? Numerous interesting questions suggest themselves. How did human beings like ourselves ever come to believe the nonsense they did about numbers? And what can have induced reputable scientists of the twentieth century A.D. to fetch their philosophy of science from the sixth century B.C.? Have the numerologists —the number mystics—been right all these centuries and the majority of thinking men wrong?

As to the origin of it all, it began some twenty-six centuries ago in the simplest arithmetic and the most elementary school geometry. None of this is beyond the understanding of a normal child of twelve. As for who may be right and who wrong, a physicist or an engineer usually is more easily seduced than a mathematician or a logician by a mathematical demonstration. Few engineers or physicists would devote their best thought to a small but incisive treatise on the unreliability of the principles of logic. It took a mathematician to do that. Logic in its most reliable form is called pure mathematics; and though mathematical reasoning, like any other, has its drastic limitations, it is still the most powerful known. But because mathematics seems to create something out of nothing, whereas it does not, superhuman

powers have been ascribed to it, even by logicians and mathematicians.

When a complicated mathematical argument ends in a spectacular prediction, subsequently verified by observation or experiment, a physicist may be excused for feeling that he has participated in a miracle. And when a skilled mathematician astounds himself with a discovery he had no conscious intention of striving after, he may well believe for a few moments as Pythagoras believed all his life, and may even repeat—after the eminent English mathematician, G. H. Hardy—the following confession of faith. "I believe that mathematical reality lies outside us, that our function is to discover or *observe* it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations,' are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards. . . ."

On coming out of the daze at his own brilliance the average twentieth-century pure mathematician might begin to doubt at least the practicality of this Platonic creed, especially if he happened to be aware of what has taken place in the philosophy of mathematics since the close of the nineteenth century. The doubter might even agree with the distinguished American geometer, Edward Kasner, that the "Platonic reality" of mathematics was abandoned long ago by unmystical mathematicians, and marvel with him that rational human beings could ever have believed anything of the kind. As he puts it, "We have overcome the notion that mathematical truths have an existence independent and apart from our minds. It is even strange to us that such a notion could ever have existed. Yet this is what Pythagoras would

have thought—and Descartes, along with hundreds of other great mathematicians before the nineteenth century. Today mathematics is unbound; it has cast off its chains. Whatever its essence, we recognize it to be as free as the mind, as prehensile as the imagination."

It is not for us to judge between the two schools of thought. We note only that each of the authorities cited as a witness to the truth of mathematics published his conclusions in 1940. Even in a court of law it would be difficult to find a sharper disagreement between competent experts. A like irreconcilable difference of opinion severs the modern Pythagorean scientists from those of the older school, who still believe that reliable knowledge of the physical universe cannot be attained without observation and experiment.

My sole objective in the following chapters will be to see how these differences of opinion came about. Though the theme is number, no mathematics beyond the simplest arithmetic is required for following the story. An occasional allusion to some obvious statement about straight lines, such as young children are taught in school, need not terrify anyone if it is called geometry. The important things are not these trivialities of a grade-school education. What matters is the weird nonsense people no less intelligent than ourselves inferred from these trivialities. To prevent our excursion into the past from degenerating into a journey through a valley of dry bones, we shall become as well acquainted as we may with the great men primarily responsible for our present widely divergent opinions. The majority of the men cited are famous and their major contributions to civilization well known. The aspect of their work which is of interest here may be less familiar, though it was no less important for them than the

things for which they are commonly remembered. A few names may be new to some. They are only about ten out of hundreds who left their mark on number mysticism and all that it implies for our own attempt to think straight.

Those who have had no occasion to examine for themselves what the ancient lore of numbers has done, and is doing, for their thinking habits, may be interested to linger for a while at the principal shrines where the magic of numbers paused on its way from the past to the present. Time and the continual changes in the meanings of words have confused the historical record till the hard core of arithmetical fact at the center of some ancient wisdoms is not always evident at a casual glance. Much of the influence of such apparently trivial statements as "three and seven make ten" on philosophic, religious, and scientific thinking is crusted over with the symbolisms of outmoded attempts to fabricate a meaningful image of the material universe. Ambitious and inspiring as such efforts may have been, they are far surpassed—at least in ambition—by some of the earlier struggle to explain human values in terms of numbers. Virtue to the highly imaginative Pythagoreans of antiquity was one number, vice another; and the elusive concepts of the True, the Beautiful, the Good were sublimated into "Ideal Numbers" by no lesser a metaphysician than Plato. And if it seems strange that Pythagoras should have believed that love and marriage are governed by numbers, we have but to observe the like today.

Step by step the immemorial magic of numbers has kept pace with unmystical science all down the centuries. If the patient investigation of numbers has aided the development of science and furthered such enlightenment as science can give, it has also perpetuated older beliefs that but few tolerant

men would call enlightened. Once of scientific certitude, these stubborn superstitions long ago ceased to have any meaning for the literate. But the belief that number is the ultimate answer to all the riddles of the physical universe, though subtly disguised, is still recognizable in the refined mathematical mysticism of the modern Pythagoreans. Our principal concern will be to retrace the main steps by which this overwhelming conclusion has reached the living present from a past so remote that only rumors of its existence survive.

To anticipate slightly, three types of mind have been lured into comprehensive theories of life and the universe by the deceptive harmonies of numbers. Contrary to what common sense might predict, mathematicians were not the first but the last to take numbers seriously, perhaps too seriously. Behind every mathematician in the dawn of numerical thinking was a scientist, and behind every scientist a priest. The scientist may have been only a primitive astrologer who read into the wanderings of the planets more than any astronomer has yet discerned. Still, he was a scientist in that he attempted to reduce his crude observations of nature to a rational system.

To the priest looking over the scientist's shoulder the irrepressibly prolific numbers repeated a familiar tale. He and his kind had known for centuries that the most potent of all magics resides in numbers. But it was not until the common run of mankind had accepted number as an almost universal convenience in astrology, in trade, in agriculture, in astronomy, and in primitive engineering, that men who today would be recognized as mathematicians arrived and began to study numbers for their own sake. Their contribution to the stock of reliable knowledge provided more imaginative men with an inexhaustible store of curious relations between numbers

to interpret as they would. The outcome was a golden age of Greek philosophy.

By the time some of the more versatile interpreters had elaborated their theories of truth and the material universe, the plebeian ancestry of the noblest doctrines of certain aristocratic philosophies had been forgotten. What was reputable arithmetic then became the exclusive possession of mathematicians and scientists. Simultaneously the old magic of numbers passed into the hands of sincere but deluded zealots, whose intentions no doubt were good, but whose sacerdotal juggling with the trivialities of arithmetic was barely distinguishable from conscious charlatanry.

With the advance of experimental science in the seventeenth century the ancient magic of numbers gradually became disreputable. It then sank almost completely out of sight in philosophy, though Kant in the late eighteenth century had some of it, and the very positive Comte about fifty years later almost lost his philosophic reason in the vagaries of numerology. What remained of it throve rankly in such dubious occupations as fortune telling. But never was its less fantastic part quite dead. Then suddenly in the third decade of the twentieth century the period of suspended animation ended. Resplendent and respectable in the dazzling symbolism of a brilliant new physics, the ancient magic of numbers rose again to vigorous life. Number returned as the ruler of an infinitely vaster cosmos than all the cramped heavens Pythagoras and Plato ever imagined. Executing an abrupt about-face, the modern Pythagoreans marched back to salute their master and offer him the augmented tribute of his own.

A Royal Mace

UNTIL adequate food, clothing, and shelter are reasonably secure only the hardiest souls have leisure to ponder over man's place in the universe. It is not surprising therefore to find the utilitarian motive predominant in by far the greater part of the earliest work in numbers of which there is definite record. The Egyptian farmer of five or six thousand years ago, for example, needed to know when the annual inundation of the Nile valley could be expected, and for this a fairly reliable calendar was a necessity.

Even the crudest calendar presupposes a familiarity with numbers far beyond that attained by all but the most advanced of primitive peoples. The art of counting was not perfected in a day, and many a semi-civilized tribe has stopped short of ten in its efforts to enumerate its possessions. For such peoples all numbers greater than half a dozen or so are indistinguishable from one another and blur in an unexplored vastness. They are of no more practical importance to a homeless nomad than infinity is to a Wall Street accountant.

Instead of the modern mathematician's "infinity," the wise man of a small tribe groping toward counting contents himself with an equally nebulous "many." This is sufficiently

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accurate for his magical predictions: the margin between starvation and plenty is as adequately covered by the difference between six and ten as it is by the vaster unknown between ten and fifteen. By eye rather than by intellect the seer who is just a shade more observant than the herdsman senses whether the tribe has enough; it is immaterial whether it has too much.

✗ It is unlikely that we shall ever know when, where, or how human beings first learned to count with the unthinking facility of a civilized child of seven. Nor is it probable that we shall discover what people first mastered the art of counting in all its freedom.

Admitting only tangible evidence, we can assert positively that by 3500 B.C. the Egyptians had far outgrown the primitive inability to think boldly in terms of large numbers. A royal mace of about that time records the capture of 120,000 human prisoners, 400,000 oxen, and 1,422,000 goats. These very impressive round numbers suggest one of two things. Either the victorious monarch had an active imagination and an inflated ego, or the Egyptian tally keepers had learned to estimate large collections by multiplying the number of individuals in an accurately counted sample by the guessed total number of such samples.

But even this remarkable feat and others almost as spectacular do not indicate that the Egyptians of 3500 B.C. were aware that the sequence of numbers 1, 2, 3, 4, 5 . . . is indeed endless. They may have believed subconsciously that it is always possible to conceive a number greater by one than any imagined number, but they did not put their belief on record. For anything we know to the contrary the Egyp-

tians may have believed that the numbers 1, 2, 3 . . . come somewhere, sometime, to an end. A cast of thought more subtle than theirs had to evolve before the concept of an infinite collection could become a commonplace of mathematics and philosophy.

The 120,000 prisoners, 400,000 oxen, and 1,422,000 goats of the royal mace do, however, reveal a fact of cardinal importance in the evolution of numbers. We who learn to count glibly before we can read may have overlooked the only thing of deep significance about numbers in the entire process. This must have taken almost superhuman penetration to see when it was first observed; and it is a fair guess that very few of even the most alert observers would notice it in the conqueror's boastful cataloging of his loot. As with some other fundamentals of mathematics and science, the difficulty with this one is its apparently trivial simplicity—once it is pointed out.

Looking over his human captives and the rest, what could the victor say about each of the three groups that would be true for all? He might have observed that all three were composed of living individuals. Probably he did; but if so he did not consider the observation to be of sufficient importance to merit preservation on a ceremonial mace. Actually what he noticed and recorded was that all three of the groups—human beings, oxen, goats—could be compared by one and the same process. They could all be counted.

If this seems too trivial, we may try to imagine some characteristic other than the number assigned to each of the groups that would be equally significant and as potentially useful. The required characteristic is to be wholly independent of the natures of the individuals composing the several

groups. Perhaps this is too easy; to state the problem in all its generality, what is it that all of several collections of any material things whatever have in common? Each collection is countable. Moreover, as the conqueror doubtless knew, it makes no difference to the final tally in what order the things are counted, or whether the counting is done by ones, or by sevens and ones, or by tens and ones; the outcome will always be the same. The conqueror's magicians might convince their lord that one mace could become two. But they could not have shown him 1,422,001 goats by counting only 1,422,000.

The deceptive simplicity of counting conceals the very things that have made it useful and philosophically suggestive. To give them names, these may be called the universality and the invariance of the numbers generated by counting. Universality—the always true, the always relevant—has been a goal of many philosophies. Invariance—changelessness in the midst of change—has been the quest of more than one religion, and in our own century has helped to codify the laws of the physical sciences. To take an example from everyday experience, any five persons, say, meet and part. Whatever they may do, however widely they may scatter over the earth, and however diverse their fortunes, the "five" that numbered them remains unchanged. It is independent, as nothing else in their lives may be, of the accidents of space and time. Moreover the same "five" would enumerate the individuals in any group of any five things whatever.

Commonplace to us, the universality and invariance of numbers were many centuries beyond the imagination of the stewards who counted the captives. Numbers were useful to them, and that was all they needed to know in order to sur-

vive and prosper. The origins of counting were so remote, and their own civilization so far advanced, that it probably never occurred to them to ask what a number is, or to speculate on how human beings ever chanced to invent numbers. All such troublings of the spirit were thousands of years in the future. Not even the inquisitive Greeks asked explicitly what numbers are, though Pythagoras and his followers occasionally spoke of them as if they were alive.

The other question, as to who invented numbers, may be improperly posed. It is conceivable that numbers were never deliberately invented by any one man or group of men, but evolved by almost imperceptible stages, somewhat as language is believed by some to have developed from meaningless cries. Somewhere, somehow, human beings may have drifted into the habit of using numbers without knowing what they were doing. Nonetheless, the numbers 1, 2, 3 . . . exhibit some of the marks of sudden inspiration and conscious invention. The most significant of these are again connected with the universality and invariance of numbers. Although nobody knows whether such a thing ever happened, it is tempting to imagine that some nameless genius quite suddenly perceived that a man and woman, a stone and a slingshot, a dream and a sunset, and in fact any couple of things whatever, are all alike in one respect and only in one: their "twoness." From there to the conception of the number two itself was a gigantic stride, but some man must have taken it centuries before the King reviewed his captives.

Lest all this still seem too easy, let us accept the number two as the commonplace it appears to be, and ask ourselves what two, considered as a number independently of its uses, "really is." In short we are to define the "number" two in

a manner acceptable to at least some (but not all) twentieth-century mathematicians. A similar definition is to hold for any natural number.

It is not easy. Between the counting of the 1,422,000 goats and a reasonably satisfactory definition of two, there is a blank of about 5500 years in which neither mathematicians nor logicians could satisfy themselves what two is on its own merits. With the caution that finality is the last thing any instructed mathematician strives to attain in mathematics, we shall merely state the definition. Two is the class of all those classes of things which can be paired off, one-to-one, with the members of any couple of things. "Class" is to be understood intuitively as a primitive notion not further analyzed. The apparent circularity in "two" and "couple" is only accidental and can be avoided. Thus the natural "number" two is a "class"; and similarly any natural number is a class.

Without attempting an analysis of this rather recondite definition, we note that when pondered and understood, it captures what eluded the first man who observed that all such collections as a husband and his wife, a dawn and a death, a bird and a thunderstorm, have in common only their twoness. This observation, whoever made it, was the beginning of arithmetic. It was also the secret source of all the magic of numbers that insinuated itself into ancient philosophy, mediaeval number mysticism, and modern science.

We have noted one possible origin of numbers. In suggesting that numbers were invented we did great but unintentional violence to more than one respected philosophy of number, Plato's among them, and outraged the beliefs of many

eminent mathematicians of the nineteenth and twentieth centuries. Historically an obvious alternative has been far more widely accepted. If numbers were not invented by human beings they may—not necessarily "must"—have been discovered. Here is the parting of the ways, where knowledge ends and opinion begins.

Some mathematicians believe that numbers were invented by human beings. Others, equally competent, believe that numbers have an independent existence of their own and are merely observed by sufficiently intelligent mortals.

The difference between the two creeds is anything but trivial. Both cannot possibly make sense. It is conceivable, however, that the question "Were numbers invented or were they discovered?" is improperly posed. It may seem as meaningless to our successors as the question "Is honesty blue or is it triangular?" seems to us. But at present—until the psychologists intervene—the question about numbers seems to make as good sense to us as some others which can be answered unambiguously. For example, "Was America discovered in 1492 or was it invented then?", or "Did Watt invent the steam engine or did he discover it?"

Even superficially these four specimen questions are of different types. Though the one about honesty has the grammatical form of a meaningful question, it is merely a nonsensical string of words. The one about America could be quickly settled, except perhaps in a metaphysical debating society, by accepted methods of evaluating historical evidence. The question of Watt and the steam engine might be resolved similarly. Then some thoughtful philosopher might remark that the eternal structure of the physical universe and the constitution of the human mind necessitated the inven-

tion of the steam engine sooner or later in the destined un-
folding of history. Without laboring the point we note that
a case might be made out for Watt as part inventor, part
discoverer. It is even possible to make some kind of sense
of the assertion that the steam engine was waiting to be dis-
covered ages before the solar system came into being. Watt
would then be a mere observer of the already existent.

The question about numbers—were they discovered or were
they invented?—cannot be disposed of by any such means
as suffice for the one about America. Whichever answer we
favor is determined largely by our emotions. For plainly the
question is unanswerable by any objective or documentary
test, and yet it is not, apparently, nonsensical. In this it re-
sembles several profounder questions concerning man's rela-
tion to the universe that have exercised philosophers, theo-
logians, and scientists for many centuries. Those who would
say numbers were discovered might agree that man is the
noblest work of God. Those favoring a human origin of num-
bers would be inclined to retort that man almost invariably
has made his gods in his own image.

It is not necessary to take sides in this age-old controversy.
Our only concern here is to observe certain phases of it down
the centuries, and to note how deeply men's beliefs concern-
ing the Platonic reality of numbers—their existence as supra-
human "entities" beyond man's interference—have influenced
their beliefs in other fields far distant from mathematics and
perhaps of greater human value. Whether the question "Were
numbers invented or were they discovered?" is answerable
or unanswerable, or whether it is meaningful or improperly
posed, its impact on the development of rational thought
more than once has been decisive. Emotional or rational

attempts to answer it continue to generate controversy, if nothing more profitable. It is the oldest and the simplest of all questions regarding the nature of mathematical truths. History gives no universally accepted answer to it; science, it is hoped, may.

Instead of trying to come at the origin of numbers by conjectural reconstructions of the history of our race, psychologists have sought the same goal by imagining the early development of the individual. Counting becomes a possibility to the future arithmetician when, as a very young baby, he falls out of his crib or bumps into a chair. For the first time in his life he then senses the "not-I." The "I" and the "not-I" are the matrix of all plurality. It may not be too fanciful to see in this shattering recognition of a hostile "not-I" the subconscious beginning of the evil associated with the number two by all number mystics from the ancient Pythagoreans to the theological numerologists of the Middle Ages. Two, the "Dyad," the "not-One" invariably is represented as unstable and bad, as deceptive indeed as a two-dollar bill. The number-wise Dante (thirteenth century), for example, argues that the Empire should be "unified" because "being one" appears as the root of "being good," and "being many" the root of "being bad." It is for this reason that Pythagoras puts "One" on the side of good, and "Many" on the side of evil. Dante might have added that Plato followed Pythagoras in this respect and that each may have been recalling the subconscious memories of his infancy. Unless the future number-mystic is also a born solipsist he will learn very early that he is not the all-powerful, all-knowing One and Eternal Monad.

Further painful encounters with tables instead of chairs may lead to the perception of "not-chair." The infant's loving parents and the not-so-loving family cat impress further distinctions on his raw and tender consciousness. But unless the infant is to be a great mathematical philosopher, he will not intuit his parents and the cat as sharing anything universal with the inanimate trinity of two chairs and a table. Indeed he will probably never discover (or invent) "3, 4, 5 . . ." by himself, but will have to be taught them by his parents. From whom did his parents learn the numbers? From their parents. And so on, back to savagery.

At this point the psychoanalysis of number becomes somewhat less sure of itself. From whom did the savage learn? His parents stopped at the number six. Did the genius of the tribe invent the "seven" he used to count the arrows his father was unable to enumerate? Or was "seven" waiting to be summoned from the realm of Eternal Being? And will it still be there when the human race is extinct, ready to be rediscovered by some future species of intelligent animals? How much of "number" is created by the human mind—or by human behavior—and how much is self-existent and only observed? It will do the practical man little good to say that only a metaphysician would ask such questions. The historical fact is that numerous impractical men not only asked these questions but struggled for centuries to answer them, and their successes and failures are responsible for much by which the practical man regulates his life in spite of his impatience with all metaphysics.

As usual in such inquiries the favored answer is an inconclusive compromise. Experience teaches the savage that number is a reliable label for distinguishing objects whether like

or unlike. Once he has perceived the difference between one thing and many, the savage is compelled (by what?) to continue through "three" things to "four" things. and so on for as far as may be profitable. Only at a much later stage, when civilization is a habit, do fairly general conceptions of numbers emerge. At some intermediate stage such arithmetical facts as $4 = 2 + 2, 4 = 1 + 1 + 1 + 1$ must have been apprehended at least intuitively. Any conception of numbers which contradicted these basic facts of arithmetic as we know it would be rejected as too clumsy for use.

Though abandoning the main question unanswered. this compromise has the double merit of leaving open two doors. one to naturalism. the other to supernaturalism. After the first step hesitation was no longer possible. The scores of mystics, philosophers, and mathematicians who chose the second door beheld the vision of number as a divine creation. Some even saw number as the power to which even the gods must bow. Those who preferred the way of naturalism found nothing superhuman. Their negative reports were largely ignored, and they themselves achieved no great popularity. The few independents who refused to enter either door and maintained their open minds had almost no support.

The next significant historical episode after the royal mace of 3500 B.C. concerns the Babylon of fifteen centuries later.