

Hans Kayser

# HARMONIA PLANTARUM

*The Harmony of Plants*

---



First published 1943 by Benno Schwabe & Co. Verlag, Basel, Switzerland  
Translated by Ariel Godwin  
2008

Translation Copyright © 2010  
Sacred Science Institute

First English language edition published by Sacred Science Institute 2010

Originally published in German under the title Harmonia Plantarum  
Benno Schwabe & Co. Verlag, Basel, Switzerland, 1943

All rights reserved. No part of this book may be reproduced or utilized in any form,  
or by any means, electronic or mechanical, including photocopying, recording, or by  
any information storage and retrieval system, without permission in writing from  
the publisher.

SACRED SCIENCE INSTITUTE  
POST OFFICE BOX 3617  
IDYLLWILD, CA 92549-3617  
[WWW.SACREDSCIENCE.COM](http://WWW.SACREDSCIENCE.COM)  
[INSTITUTE@SACREDSCIENCE.COM](mailto:INSTITUTE@SACREDSCIENCE.COM)  
(951) 659-8181 ☎ (800) 756-6141

## Contents

<i>Translator's Note</i>	v
<i>Dedication</i>	vii
<i>Introduction</i>	ix
Part 1.	
<i>The Form of the Plant</i>	1
The Overall Form	3
Digression on Goethe's <i>Metamorphosis of Plants</i>	44
The Form of the Leaf	52
Bifurcation	63
Leaf Collectives	97
Leaf Positioning	99
The Flowers	115
Part 2.	
<i>The Functions of the Plant</i>	141
Life	143
Sex	150
Nourishment	155
Chlorophyll	157
Chemistry	159
Geotropism	165
Growth	167
Spontaneous Generation	168
Aspects	173
Part 3.	
<i>The Harmonic Value-Forms of the Plant</i>	175
Preliminary remarks	177
Discontinuum	178
Hierarchy	181
Rhythm	185
Polarity	190
Symmetry	194
Resonance	199
Spirals	201
Senarius	207
Directions (Tropisms)	209
Part 4.	
<i>The Nature of the Plant</i>	211
<i>Afterword</i>	227

## TRANSLATOR'S NOTE

The purpose of this translation is to make Hans Kayser's work accessible to the English-speaking world. Kayser, throughout his life, was first and foremost a harmonist: a scholar dedicated to the study of the fundamental laws that govern music and their applications in other disciplines. Consequently, this book should be approached above all as a harmonic work, and not as a botanical study, even though Kayser's knowledge of the natural sciences was impressive. In the decades since this book was written, many new discoveries have been made in botany, and many changes have taken place in taxonomy. To contemporary readers versed in the corresponding subject areas, some of the botanical content of this book will appear outdated. But its harmonic content—which is what gives this book its great value—is still current. The laws of harmonics are eternal, and Kayser understood them to an unparalleled degree.

Hans Kayser was, among many other things, an accomplished graphic artist. In some of the more complex diagrams in this book, especially those showing the partial-tone coordinates, it was judged aesthetically inappropriate to convert the German names for the notes (e.g. *fis* for *f*#, *h* for *b*) to the English names. For readers who wish to explore these diagrams in depth, we provide the following reference table:

<i>German</i>	ces	c	cis	des	d	dis	es	e	eis	fes	f	fis	ges	g	gis	as	a	ais	b	h	his
<i>English</i>	<i>c</i> ♭	<i>c</i>	<i>c</i> ♯	<i>d</i> ♭	<i>d</i>	<i>d</i> ♯	<i>e</i> ♭	<i>e</i>	<i>e</i> ♯	<i>f</i> ♭	<i>f</i>	<i>f</i> ♯	<i>g</i> ♭	<i>g</i>	<i>g</i> ♯	<i>a</i> ♭	<i>a</i>	<i>a</i> ♯	<i>b</i> ♭	<i>b</i>	<i>b</i> ♯

However, in the text of the book and in the less intricate diagrams, the note names have all been converted to English.

Ariel Godwin  
Columbus, Ohio  
2008



"All forms are similar, and none  
is the same as the others,  
And so the chorus tells of a  
secret law,  
Of a sacred mystery..."

GOETHE

"Where once with plants  
my father lovingly taught me..."

HÖLDERLIN

## *Dedication*

To you, dear father, I dedicate the leaves of this book. For thirty years now you have been in that world which, from here in this land of gloomy dreams, you always sought and never found, yet forever longed to reach. Music and plants—for you they were the two openings through which divine light shines into our souls. The former as the herald of eternal norms, the latter as the master of unchangeable laws in emerging life: tone and form, to you, were a guarantee that there was meaning in being a resident of this world, and that the ultimate answer to our yearning to belong could be found through listening.

So many times I went with you to look for plants, even as a small boy. Hardly a valley, peak, or cliff of this florally rich alpine corner of Germany remained unknown to us. But only sometimes—and then only in small amounts—did we bring a crop home with us. The rest remained in our heads; and even today, like treasured old paintings, the images of those countless clusters of beautiful and rare flowers, bushes, and shrubs are preserved unfading in my memory.

The nailwort clinging to the rocks, the fragrant sunroses growing beneath overhangs, the elegant lady's slippers, the great yellow gentians and many other smaller gentians blooming on the peaks, the pasque flowers as the first sign of springtime on the ledges, the Turk's cap lilies, the fly orchids, bee orchids, and bumblebee orchids in the clearings, with all their variations, the dozens of other orchids—I can still smell the intoxicating scent of the hyacinth—and more orchids on the valley slopes, bellflowers growing as tall as a man, foxgloves in shady hollows on the slopes of the Danube valley: these are only names, only sounds for the readers of this book—but for the boy whose youthful mind was led so wisely and lovingly toward his first sight of the eternal and beautiful in nature, they are an unforgettable and unfading experience.

And often, when we returned home tired, once you had closed down the office, another realm lay awaiting us, an inner realm, that of music. At the age of forty, in that little town so far away from all musical centers, you had learned to play the viola, and you sat me down, a ten-year-old, with the cello. With other enthusiasts, we jumped rough and ready into Haydn quartets, then played Beethoven and Schubert; and even the old stone statue by the quietly bubbling fountain in the market square in front of our house must have felt a flutter in its chest at the sound of our heart-stirring melodies. It was wonderful to me then, and still wonderful to me today!

Now, after many years of wandering, after lengthy researching, pondering, erring, seeking, and finding, having settled in a new country—now the circle is closed once again: in this book I return to you, father, and in it you come alive. It was the blue flower whose sound you taught me; may the contours of its silhouette illuminate the following pages...

**P**lants! In an eternal harmony, the life of the plant, and with it the primal beginning of all life, enters external existence: light, water, and earth weave the threads of the world in infinite forms.

The light brings warmth. Restoring matter to life, the intervals of its quanta vibrate through the atomic formulas of the protoplasm. It conjures the framework, the support, color and form, out of the earth into open space. It uses water for the metamorphosis, the building of the form in time. It is the tonic, the light-tonic, its partial tones forming that eternal harmony of life, and further modulations and variations are determined by the order lying within it.

These are the foundations for the “outer expressions” of the first living beings, the plants; the foundations for their growth, their materiality, their form. The way from there to the “inner expressions”—which is the harmonic way—does not require the above musical metaphors. Every perceptive person “loves” plants. This love for plants—meaning, of course, this psychical relationship with them—traverses a whole series of aspects. Not only the history of botany, but also the position of humans toward plants itself, reveals the manifold nature of this psychical relationship. Even its most external form, simple classification, comparable with naming the stars in the sky, brings us to a certain intimacy with plants. By giving them names, we learn the differences; we gain an idea of the individual, the specific, the unique, and in noting the names of the plants, we create their image within ourselves. In various ways, we can deepen and intensify this tone of the plant in us. The laws obeyed by growth, the architectonics of form, the peculiar logic of chemical structures, the miracle of the pollination mechanism, the ecology of the plant world, and many other things: even amateurs will feel deep resonances from these insights between themselves and the world of plants. And how much more of this will the scientist, the botanist, feel if he has retained the glow of enthusiasm and not hardened himself into stifling, soundless, bloodless clinicality.

Thus there are enough psychical relationships between us and plants, whether they are merely on the level of love and the ability to feel enthusiasm at the beauty of plants in general, or in the direction of an inner understanding of the countless important and interesting questions involved. Love and enthusiasm are not the only languages of the heart; the perception, researching, seeking, and solving of problems must also come from the depths of the mind, must be rooted in the soul, in the heart, if the results of research are not to turn out as a lifeless mechanism of systems, as mere mental acrobatics.

What is the position of *harmonics* in human perception and human knowledge?

If we answer: “Harmonics is a science that understands things with the heart and perceives them with the intellect,” then we have expressed the characteristics of the harmonic approach, namely tone-number. This approach involves perceiving and thinking, heart and

intellect, tone and number, all unified, and from it the entire structure of harmonics is built.

What meaning this harmonic approach has, not only in the modern world but also in earlier epochs, can be briefly expressed thus: harmonics makes at least an attempt to build a two-way bridge, a “turnpike,” a possibility for understanding on a universal human and psychical basis, in opposition to the dissociation, the individuation, indeed the sharp separation observable in all eras between human means of expression such as religion, philosophy, art, and science. But this harmonic attempt does not take place merely on the level of certain universal connections. It sounds things out by means of forms present in us, which we perceive as “correct” simultaneously with our soul *and* our intellect. By means of these “value-forms”—which we verify for ourselves, rather than filtering and abstracting them from an allegedly “objective” condition, external to us and fundamentally accomplishing nothing—and which are, perhaps, the only passport available for the spiritual traveler of modern times—we set forth upon the search for the sounds and melodies that we have heard within the depths of ourselves. In our wanderings we do not take possession of cities or countries, we do not impose our experiences upon anyone. But we are sure, wherever we go, that we will find, friends, brothers, and sisters of the same disposition. In this disposition, our hearts speak, and we endeavor to understand time, past, and future, and to rebuild what we have heard in ourselves according to their forms; and thus, as true humanitarians, we belong to ourselves, indeed the world belongs to us. Only then are we citizens of this world and guarantors for this world.

In observing plants, the inquiring spirit comes face to face for the first time with the powerful and enchanting, imminent and alluring question of *life*. From the abysmal depths of the dark and mysterious things into which we humans plunge, in which we are enmeshed at every turn, this life glows as the great puzzle whose solution has been, since ancient times, an object of human pondering. Here, we shall attempt to give an interpretation of this puzzle from a harmonic standpoint—a drop of understanding in the ocean of nescience that surrounds us.

Here we must make some introductory remarks regarding a specific topic: the characterizing of the harmonic position on biological questions in general.

In recent times, the study of life, i.e. biology, has been pervaded by a lively and sympathetic approach, whose developers (Uexküll, etc.) call “environmental theory.” This theory completely avoids “anthropomorphism,” as is stated, for example, in Brehm’s *Life of Animals*, and endeavors to examine the environment of a living being as much as possible from the point of view of this being itself, rather than from the point of view of our human interests. Fundamentally, this is a matter of an old tendency toward objectivization, to which the exact sciences have accustomed us for a long time: the objectivizing of optics in the “anthropomorphic” study of color and light, of chemistry in alchemy, of astronomy in astrology, and so on. Obviously, the efforts at emancipation that have emerged relatively late in biology are in a certain sense able to illustrate the question of life in a more pure, abstract way than previously, especially in the “humanizing” representations common to popular

scientific literature.

Upon closer inspection, however, one must wonder whether this alleged change in human criteria and turning toward the environment of a given being can be correct in terms of the theory of perception.

The life of a certain pin-head sized tick is often cited as a borderline case of an “environment” not comparable with human conditions. It affixes itself to warm-blooded creatures; their blood gives it its ability to reproduce. Although blind and deaf, it has the instinct to climb onto flexible branches and to wait there, sometimes for as long as ten years (!), for a warm-blooded animal to brush by. What does this creature’s paltry world have in common with ours? Seemingly nothing at all. It lives in eternal darkness and eternal silence. And there it must find its way, live on, and reproduce.

And yet: this creature is *alive*! Only a very few factors, admittedly, determine its perception of the world. It must have a sense of warmth, since it attaches itself only to warm-blooded beasts; it must have a certain appetite, namely for blood—a sense of taste! It must be able to distinguish shrubs and grasses from hard wood—a sense of touch! And finally, it must have a goal-oriented instinct—teleology—otherwise its process of life could not take place in this manner. Not to mention its sexual activity and its other abilities and behaviors hardly even observable to us.

Now I ask: are warmth, taste, touch, and teleology qualities that have nothing to do with humans? Are they not just as “anthropomorphic,” i.e. only judgeable according to human criteria, as everything else that presents itself to our perception? Clearly, the life of this tick is absolutely pathetic compared to ours; conversely, certain animals have specific qualities (sense of direction, sense of smell, precise judgment of time, etc.) that put those of us humans to shame. But these are all qualities with which we are familiar in ourselves, and which we can only judge from within ourselves, only assimilate “anthropomorphically” in our experience.

I gave this example, which certainly says nothing new to the environmental scholar, simply because the harmonic approach is able to simplify and clarify the question of life, as it initially emerges in plants. As has been said earlier, harmonics endeavors to find definite forms and patterns in us which both our perception and our intellect perceive as correct (tone and number!), and are thus able to verify. By means of these harmonic “value-forms,” which are established and determined in us as something psychically and spiritually understandable, i.e. as a kind of human estimation according to absolute conscience, we explore the things inside and outside us. The world within us, the world of the heart, thus gains graspable patterns; the world outside us, nature in the broadest sense, gains a perceptible, psychically pulsating tectonics. It would be presumptuous to say that harmonic value-forms could solve the world’s mystery entirely; they are only capable of an interpretation. But it is not, I believe, presumptuous to say that the silent sounds of their forms find resonances in most human and non-human realms—resonances which, arranged into a unified system and world view, have the ability to reanimate the belief, or indeed the

certainty, that there is a meaningful cycle, a meaningful structure for the entire mechanism of the world.

When harmonics, like the sciences, gives up a cheap, misunderstood anthropomorphism in favor of a pure exploration of the relevant factual domains on the basis of their own legitimacy, their specific “environments,” it is still with the awareness that we as humans can only measure and perceive things with human criteria, and that humans, albeit *sub specie aeternitatis*, are the measure of the world.

From this insight, however, the necessity arises of inspecting these human criteria anew and examining the sources from which they come. Harmonic value-forms show a new way for this.

This way is new because even though it is based on ancient traditions, it brings an entirely new perceptive approach to the modern world. In the harmonic phenomenon, tone-number, the unfortunate gap between thought and perception, between nature and psyche, is bridged *a priori*. Anyone who knows the history of philosophy and research over the last three hundred years will know the meaning connected with this. This bridging, however, is not a merging—as when H and O are made into H<sub>2</sub>O, i.e. something entirely new and different. In the aspect of tone-number, we have a synthesis that preserves both components. Tone and number do not “merge,” do not produce something new, but remain independent while at the same time united. Material vibrations (number) and tone perception (tone) each inherently belong to different worlds: the material and the psychical. And yet they are legitimately connected, and this legitimacy—or indeed, the fact that there *is* legitimacy—forms the starting point for all harmonic investigations.

The phenomenon of tone-number is the seedling from which the tree of harmonic knowledge grows. Inconspicuous, like any seedling, like any seed, its inner capabilities are not discernible. On this page before me there lies a tulip seed: a small, brown, translucent flake with a thin line, the germ, in the middle. What does this unimpressive form have in common with a tulip? If I did not know it was a tulip seed, even in my most fantastic imaginings I would not picture the miracle that this seed could produce, given fulfillment in time and space. And a microscope, even on the atomic level, would reveal nothing more than a package of molecules, giving not the slightest impression of the form and beauty of a tulip.

Just as one must give the seed the right nourishment in order for it to grow into a plant, so one must also give the correct spiritual nourishment to the harmonic seedling, the phenomenon of tone-number, if the possibilities and capabilities in it are to achieve their full development.

This analogy has brought us, unintentionally, to the periphery of the plants themselves.

In plants, as in all living beings, the psychical and the material are present in inseparable unity. Insofar as life appears for the first time in plants, the general viewpoint is that the psychical element finds its first, “lowest” realization in the phenomenon of the plant. From this point of view, the harmonic approach, in which the psychical (tone) and the

material (vibration number) are united, must have special significance in the domain of plants.

The previous results of harmonic research have shown, however, that characteristic psychical forms, as we perceive them specifically in one part of the harmonic value-forms, can also be found in the inorganic, “dead” side of nature. This “animation” of the inorganic should not be taken for hylozoism, and is not an “animation of all things,” but instead only accounts for certain psychical forms in inorganic nature, so that we might say: it is not the prerogative of humans alone, much less a special peculiarity of life, to take part in psychical forms and to grow according to psychical forms—even though it may be obvious that the psychical becomes autonomous only in the phenomenon of life, and only becomes consciousness in humans.

What peculiarity and characteristic quality the psychical takes on in the phenomenon of life, and how life becomes realized and perceptible for the first time in plants—that is the subject of the following work.

Regarding the development of a *harmonia plantarum*, just as in any other harmonic work, various ways could be taken. Being convinced that all schematism, i.e. all uniform forcing of investigations onto a path determined for all of time, can only be detrimental to the wealth of the various domains—schematism and systems are different things!—I have arranged the following work in a specific manner.

It is divided into various problems. Each problem is first treated in general as a “theorem” (in the same sense as my book *Grundriß*), which means that with the help of a testable harmonic theorem, we shall search inside ourselves for the pattern that this theorem radiates and forms as an idea in our head and heart. This pattern once found, we shall search for its meaning in the lives and the domain of plants.

We do not approach plants with feeling alone—although we all do this when we give ourselves over to the beauty of plants and flowers in nature—nor with understanding, knowledge, and interest alone, as has long been the duty of science. Instead, feeling and knowledge, sensation and thought are joined in the harmonic value-form into a unified act of perception. We flock to the teachers within us, and they give us an answer to our consciousness, a promise, that we may grasp, understand, and love the silent world of plants, since it is a part of our own innermost being.

## PART 1

### *The Form of the Plant*

[www.CosmoEconomics.com](http://www.CosmoEconomics.com)



When one blows into a valveless horn, or into any suitable non-holed and gently tapered pipe (such as an alphorn), the following notes are produced “naturally” where  $c$  is the horn’s keynote:



This progression, known as the “overtone series,” which Helmholtz called a “remarkable law” and upon which he based his famous work, *Die Tonempfindungen*, is indeed highly remarkable, firstly because of the simple fact that *only* these notes, and none of the ones in between, are produced by blowing normally. We call this phenomenon “quantization,” and it plays a significant role in many areas, especially in physics (the “leaping” of electrons; quantum theory). Secondly, because the first thing to emerge is a pure major triad, and after it, the notes become increasingly closer together. Thirdly, because the numeric law of this series, in which the octaves correspond precisely to the doublings of their “rank numbers”—e.g. 1  $c$  2  $c'$  4  $c''$  8  $c'''$  etc. And fourthly, because these rank numbers 1 2 3 4 etc. correspond exactly to the vibration numbers of the relevant notes, so that if the low  $c$  produces 128 vibrations per second, the overtones following it will produce twice, three times, four times 128 vibrations per second, and so on.

Upon these basic conditions a further series of interesting laws are built; they were first constructed in modern harmonics in connection with A. von Thimus’s rediscovery of the ancient Pythagorean legacy of the so-called “partial-tone coordinates” (see my *Hörende Mensch*, Ch. 1). These partial-tone coordinates were wrongly accused of being artificial, indeed arbitrary, since they incorporated the “nonexistent undertone series.” This mistake is corrected in Figure 1. As one can see, the lines connecting the identical notes to the adjacent monochord show the correct locations for the associated tone-values; therefore the system of the diagram must be correct in itself, otherwise this result would not be possible. But furthermore, this system must correspond to some pattern in our psyche, or else we would be unable to hear its realization in its correct form. Here, then, we have two indexed proofs for the correctness of the basic diagram of the harmonic tone system of “partial-tone coordinates”! Firstly the correct pitches on the monochord, this being in a sense the external, haptic proof, and secondly the internal, psychical proof, the fact that we hear this progression psychically as correct. Mathematically speaking, this is a group theory pattern, and this, incidentally, is the first instance of its usefulness becoming evident in the domain of acoustics.

## The Equal-Tone Lines and Their Realization on the Monochord

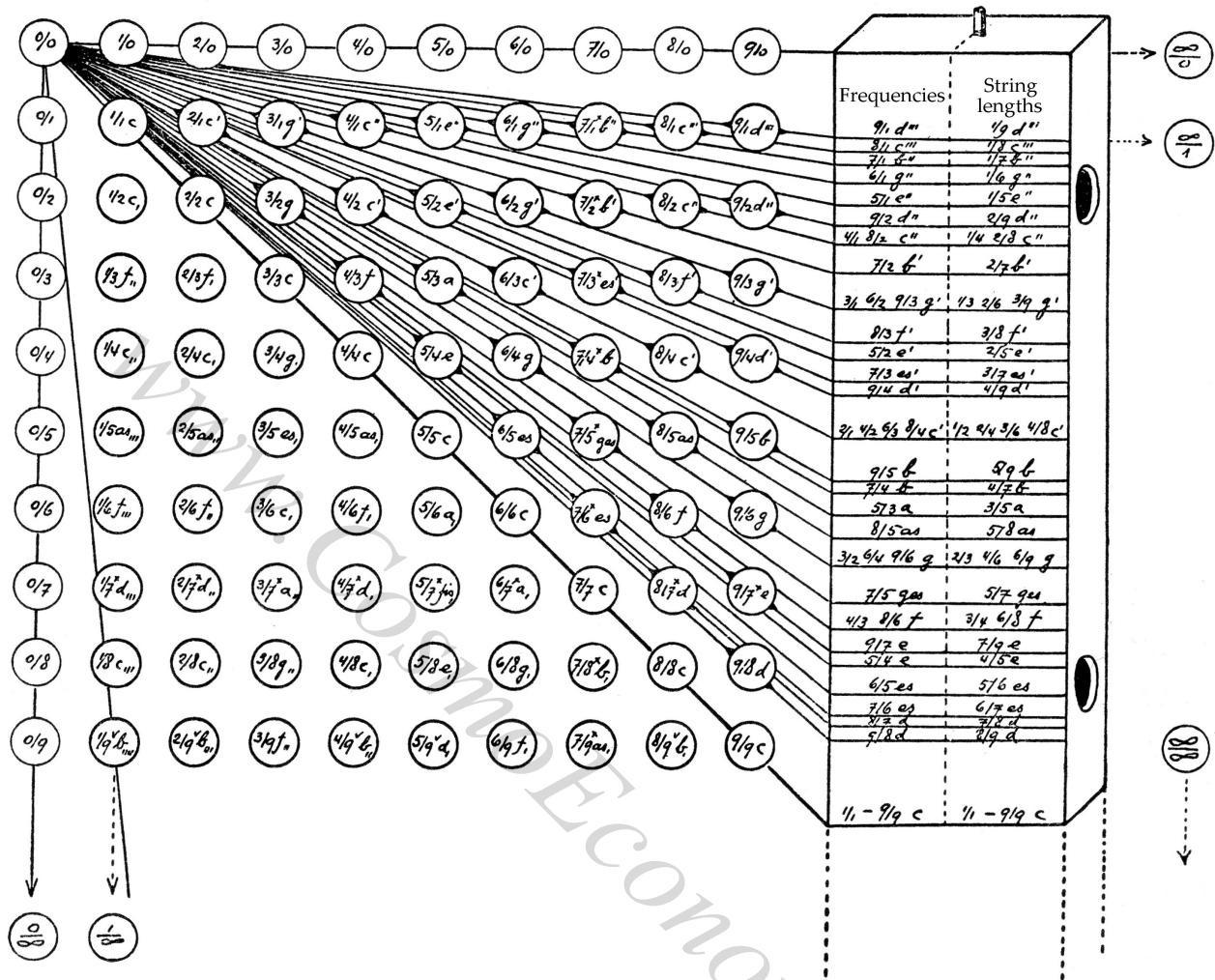


Figure 1

This diagram is based on frequencies (vibration-numbers). For the partial-tones below  $1/1$ , at the index 9 shown here, the last ray  $1/9 b b'''''$  would indicate the length of the monochord, whose string would then have to be tuned to this same note  $1/9 b b'''''$ . The distance indicated by arrows in the lower left part of the diagram, then, would have to be nine times the length of the monochord pictured in order for its tone-values to be realized.

Frequencies and string lengths are in a reciprocal relationship to one another.

The starting point of this pattern, the overtone series, is a natural phenomenon, thus existing "outside" of us, and since it is a phenomenon, we cannot influence it, only observe it. In the pattern of the partial-tone coordinates, however, our will for order plays a role, because in nature there is only the overtone series, and no partial-tone coordinates. Therefore, this starting point for harmonic tone development has been shown to us by nature in the phenomenon of the overtone series itself; it is "objective."

But there is also an inner, psychical way to reach the same goal.

When we divide a string on a monochord, or on any stringed instrument, by simple whole number ratios, then if the open string is tuned to  $c$ , we get the following tone-values:

1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	etc.
$c$	$c'$	$c''$	$c'''$	$c''''$	$c'''''$	

As one can see, this follows the same laws as above, except the number series is reciprocal to the previous one. This is because here we are using the string lengths, i.e. the spatial element, as a basis, whereas before we were measuring the vibration numbers, i.e. the temporal element. Vibration numbers and string lengths are reciprocal; their manifestation in notes yields the same values. Now, however, the division of the monochord is an experiment not performed by nature, but by us, from our own initiative. We approach this experiment with the preconceived idea that what emerges from string lengths should also emerge from simple whole number ratios. Or in other words: the internal, psychical method yields the same result as the external, objective method of the overtone series. In both cases, we obtain the partial-tone series with its peculiar laws, and thus we are further justified in ascribing both objective and subjective (internal, spiritual, and psychical) significance to the partial-tone coordinates, i.e. the harmonic tone system with its internal order and external configurations (graphic images, groups).

Before we proceed to the simplest harmonic grouping of the patterns in plants, we will say a little more regarding the fundamental value of such group analyses.

It is nothing new to apply numeric laws in themselves for the explanation, the "formulation" of conditions and forms in nature; all the so-called "exact" sciences, or at least their applications, are based on this. In modern times a specialized branch of mathematics has developed, known as group theory, in which numbers and mathematical symbols alone are not important, but rather their grouping, their arrangement. Unlike earlier, when the visual image, or rather the "geometry" or arrangement of the numbers, was merely an incidental concern, the grouping is now treated as an independent factor of crucial meaning in addition to the numbers and symbols. Matrix operations in modern physics and group theory analyses of crystal forms are only two examples of how this arrangement element has already been proven fruitful.

After contemplating, one must be amazed that the group theory element has hardly ever been applied in the natural sciences that are outside the exact sciences, such as botany, zoology, and biology (in which there are certainly many exact laws: leaf growth, the numeric laws of chromosomes, Mendelian heredity laws, etc.). All the more since the number (= the *form*) is important in group theory, and offers the possibility for the natural form to adapt itself as the rigid, formless number alone. Instead, people continue to struggle fruitlessly with difficult and awkward phenomena such as leaf bifurcation, chromosome numbers, Mendelian laws, wishing to master them with abstract numbers alone, the result being that every

formula is surrounded with a crowd of “exceptions” that rob it of its value. As far as I can see, the guilt lies less with natural science and more with group theory itself, in which—just as with modern geometry—there is a tendency to “algebrize,” which has led to the loss of the visual geometric content in both areas, and carries with it the danger of destroying the elements of group theory that are actually promising for the future. This dilemma arises initially from harmonic tone development. In terms of its pure numeric content, it embodies one of the simplest, perhaps *the* simplest, fundamental group theory number patterns. But since a tone is connected to every number, the danger of algebrization, of pure logic, is neutralized. Of course, a purely algebraic tone system could also be developed; but there would still have to be symbols for *tones*, i.e. psychical values, and this psychical verification prevents the destruction of the inner content of the harmonic “sound-images.”

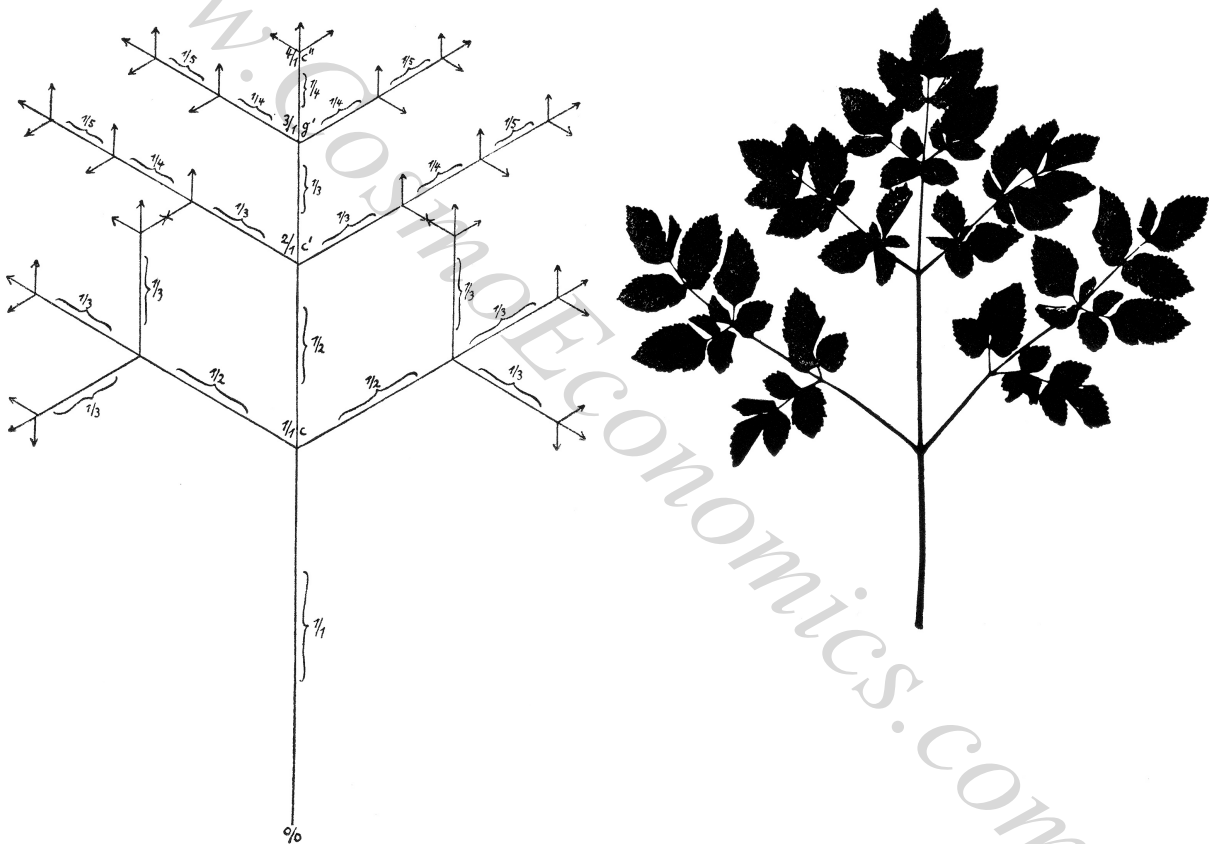


Figure 2

This last term refers to all harmonic groups and tone-number arrangements. If we may assume—and this assumption is not absurd, given the past success of group theory in physics and crystallography in terms of artificial surface ornamentation (A. Speiser), etc.—that group theory analyses may also be successful in the domain of organic forms, then we will be all the more justified in approaching the question of living forms by means of harmonic group schemata, since harmonics possesses the psychical factor that characterizes it and that is

missing from mathematical group theory: the *tone*, which overlaps the same sphere from which life itself emerges: the world of the soul.

And now, on to the harmonic analysis of plant forms!

If we use monochord division as a basis, draw the string sections with their true relative lengths as vertical axes, and allow this same division to continue on both sides at the “node points,” then the result is the diagram on the left of Figure 2, which corresponds to its plant analogy on the right.

The analogy of the plant sprig shown tells us that the correspondence is certainly not simply external. The leaf bifurcation is accomplished through a kind of node construction, like the string division; likewise, the characteristic spatial diminution,  $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ , is visible. And yet, this analysis does not tell us much more. In order to explore further into the harmonics of plant forms, we must not begin with mere string division, i.e. the simple overtone series (which we will certainly use later), but rather the complete harmonic tone system.

Figure 3 shows us the usual illustration of the partial-tone coordinates up to index 6. In order to show more clearly what is important in these modifications, the generator-tones ( $\frac{1}{1} c \frac{2}{2} c \frac{3}{3} c \dots$ ) are joined by bold lines; the identical  $c$ -values (e.g.  $\frac{1}{2} c, \frac{2}{4} c, \frac{3}{6} c, \dots$ ) are joined by thin lines; and the basal series ( $\frac{1}{1} \frac{2}{1} \frac{3}{1} \dots$  and  $\frac{1}{1} \frac{1}{2} \frac{1}{3} \dots$ ) As for the equal-tone lines, one of these can be drawn through each ratio, since at higher indexes all tones appear once again, e.g. the tone-values  $\frac{2}{3} f \frac{4}{6} f; \frac{3}{2} g \frac{6}{4} g$ . Here these lines are restricted to the  $c$ -values, in order not to make the diagram confusing and to clearly show the various directions and figurations that occur in the various diagrams.

Figure 4 is identical with Figure 3 except the  $\frac{1}{1}$  is at the bottom and the generator-tone line is vertical. Figure 5 is a combination of Figures 3 and 4, joined at the generator-tone  $\frac{1}{1}$ , according to the pattern:

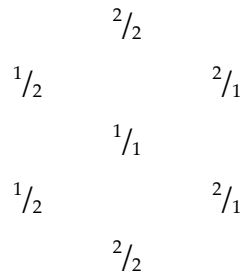
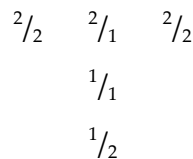
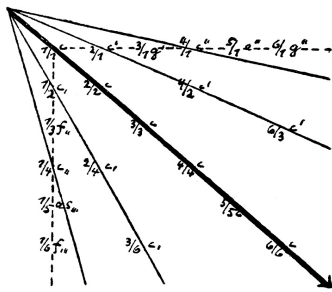
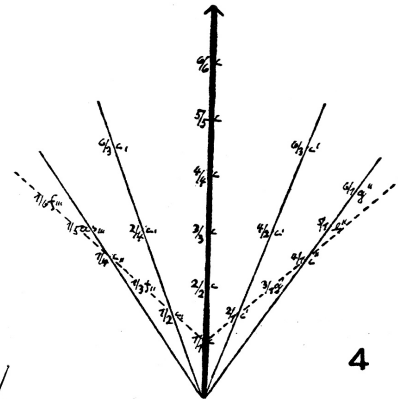


Figure 7 has this arrangement, in respective grouping:

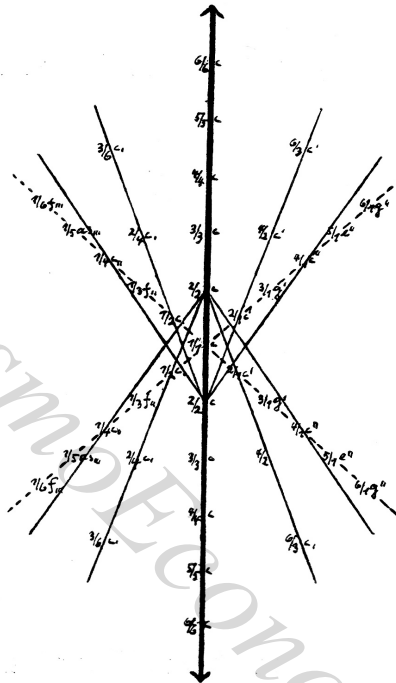




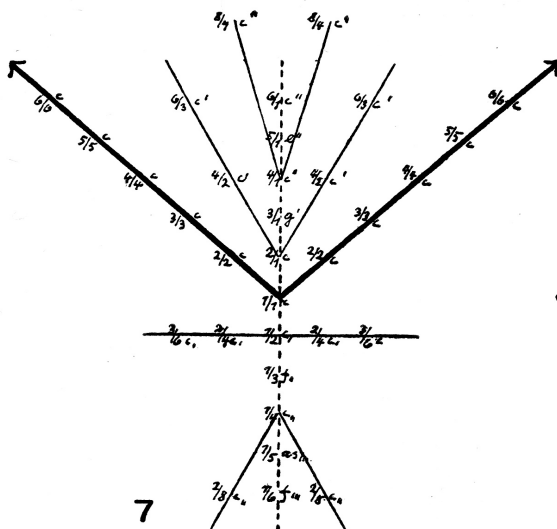
3



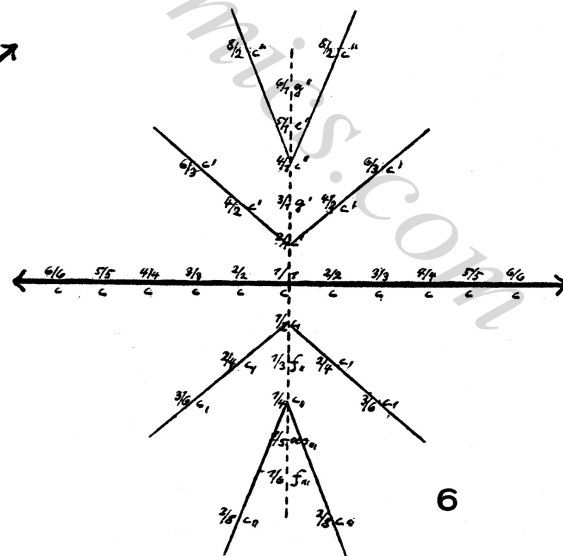
4



5



6



7

Figures 3-7

Figure 6 has the arrangement:

$$\begin{array}{ccc} & 2/1 & \\ 2/2 & 1/1 & 2/2 \\ & 1/2 & \end{array}$$

These arrangements, the actual group theory element of harmonics, is discussed at length in *Hörende Mensch*, Ch. 2, in *Klang der Welt*, p. 112 ff., and in *Grundriß*, p. 118 ff.

If we now observe Figure 5 more closely, we will notice some other things. Firstly, the central axis, secondly a kind of bifurcation caused by the partial-tone series intersecting along the central axis, thirdly a kind of directional impulse indicated by the lines connecting the identical tone-values (equal-tone lines), and fourthly the top and bottom partial-tone systems connected at the generator-tone  $1/1 c$ . In botanical terminology, these are the most important signatures of plant forms: first the stem or stalk structure, then the bifurcation, then the direction of bifurcation, which essentially determines the overall form of the plant, and lastly the polarity of roots and stem, the above-ground and below-ground parts of the plant. This correspondence between the harmonic sound-image and the “archetype” of the plant becomes even clearer when we set up the arrangement so that the basal series  $1/1 c \ 2/1 c' \ 3/1 g' \dots$  or  $1/1 c \ 1/2 c_{\downarrow} \ 1/3 f_{\downarrow\downarrow} \dots$  forms the central axis—see Figures 6 and 7. Here the generator-tone line ( $1/1 c \ 2/2 c \ 3/3 c \dots$ ) is the boundary between “above” and “below”; the upper ratios are greater than 1, which is illustrated visually by the upward-reaching, expanding above-ground part of the plant and the downward-pushing, squeezing below-ground part. Psychologically, assuming vibration numbers (frequencies), one could speak of a bright domain of plants, generated by the major impulse of the overtone series:

$$\begin{array}{c} \uparrow \\ 5/1 e'' \\ 4/1 c'' \\ 3/1 g' \\ 2/1 c' \\ 1/1 c \end{array} \quad \text{Major chord}$$

and a dark underground domain, generated by the minor impulse of the undertone series:

$$\begin{array}{c} \downarrow \\ 1/1 c \\ 1/2 c_{\downarrow} \\ 1/3 f_{\downarrow\downarrow} \\ 1/4 c_{\downarrow\downarrow} \\ 1/5 a b_{\downarrow\downarrow\downarrow} \end{array} \quad \text{Minor chord}$$

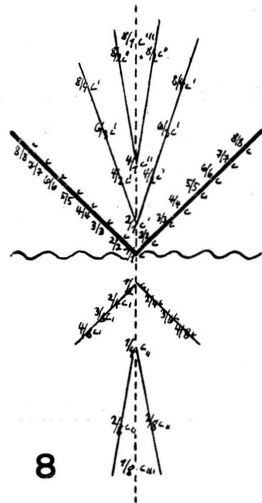
If we compare Figures 5-7, we will notice the diversity in their overall behaviors, which as we have seen is dictated by the various types of arrangements despite their having the same content. Although we are still dealing with abstractions here, these three examples already make it apparent that this element of arrangement presumably will play a major role as a morphological factor in the possibility of explaining various plant forms. But since an “arrangement” can just as well be called a “grouping,” it is evident that here a way is being shown for group theory analyses of plant forms—a counterpart to the group theory analysis of crystal forms, which has already been undertaken in mathematics (A. Speiser, *Gruppentheorie*, 2<sup>nd</sup> ed., Berlin, 1927)—in this case, on a harmonic basis, which allows for a psychical interpretation besides the merely mathematical one.

Figures 8-13 serve for the further exploration of the morphological content of the sound-image. Figure 8 shows a plant type with normal taproots and branches or twigs inclined toward the stem (or stalk). The plant type in Figure 9 shows a particularly deep taproot and branches leaning toward the earth. Figure 10, the inversion of Figure 8 in its arrangement, shows extensive roots with normal above-ground development, while Figure 11 (the inversion of Figure 9) exhibits abnormally strong root development along with an equally abnormal branchless trunk or stalk.

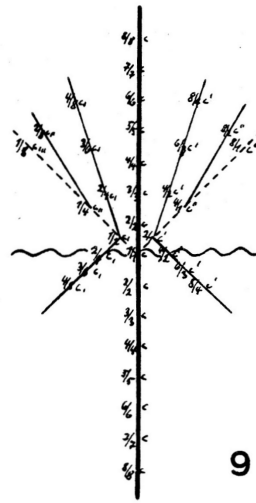
These diagrams, which are “selective,” i.e. restricted to *c*-values, become yet more interesting when one extracts other intervals from them, such as the fifth interval as the second most important ratio builder. Figures 12 and 13, “fifth extractions” from Figures 8 and 9, are examples of this. With the central axis removed, a more bushy plant type appears, and here the mystery of shrub growth lies, from the point of view of harmonic analysis, in a different selective value emphasis of the psychical content of the sound-image.

All diagrams of the above type, in fact all harmonic sound-images, must not be viewed as gauges whose number ratios can be found here and there in nature. The value lies in their “faces,” their external morphological and internal psychical content. Despite this, they will always remain closed to the reader who does not redraw them himself, coming upon new paths and ideas in the process. This working along with the images is connected with an arcane inner sense of the order of the material of harmonic laws; all those who have gone through the training of harmonic diagrams know about the remarkable charm, the compelling inner normature, which enchants brain and soul alike, spurring them ever onward to new investigations. At the beginning of Chapter 1 of my *Hörende Mensch*, I urged the reader toward collaboration and not merely “reading,” and offered the means and methods; at this place, I can only repeat the demand.

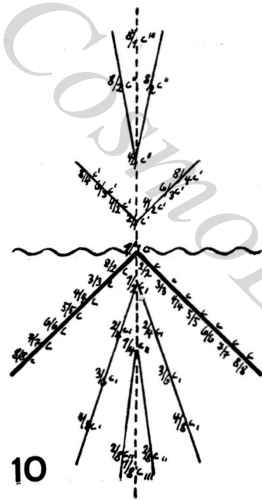




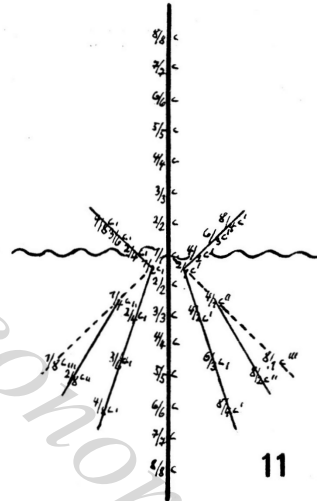
8



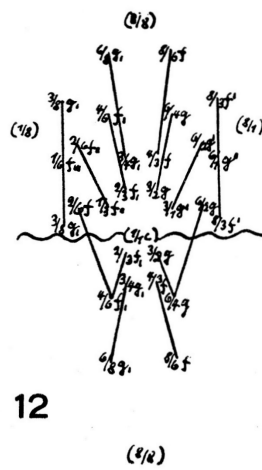
9



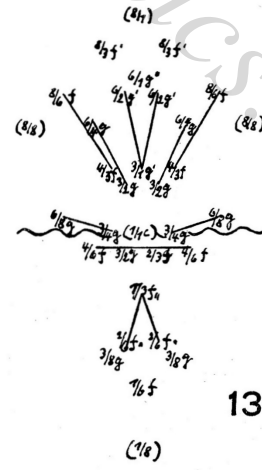
10



11



12



13

Figures 8-13

Reviewing the above diagrams, one will imagine that they represent only a part of a great number of further possible sound-images. These possibilities are limited by the arrangement and not boundless. Also, they move entirely within the "binary" division scheme. But here, we have found the possibility for a harmonics of the fundamentals of plant forms, as well as an idea of the way from these ideas to the variability of the primal plant form. Furthermore, looking back again at these diagrams gives us three new ideas: "indexing," i.e. the limiting of forms, and "selection of preferred ratios," i.e. the inner formation and finally the combination of two partial-tone coordinate systems for all biological sound-images. Figures 3-13 are developed up to either index 6 or index 8. Obviously, the diagrams could be developed up to a much higher index, which would result in ever-increasing differentiation, but also complication for the overall picture. The "index," then, always means the limitation of the form, and if one allows a diagram to run through its indexes, splitting it up into separate diagrams, then the meaning of this indexing as a progression of various stages of harmonic development will become clear. In *Vom Klang der Welt*, p. 118 ff., I developed and discussed the sound-image of a basic biological form through all indexes up to index 16, and later we will draw a few indexes from the logarithmic "sound-image" of the archetypal plant. The true meaning of the plant index will first appear from the logarithmic illustration of the sound images; here we are merely outlining the fundamental importance of this index in itself.

By contrast, the theorem of the "selection of preferred ratios" should become clear immediately from the above diagrams. It is an actual harmonic theorem in that mathematics, when speaking of "preferred ratios," must treat them as unexplainable conditions ("God created the whole numbers," "the importance of the first whole numbers," etc.), whereas harmonic selection is completely psychically determinable and consequently explainable. The reason why the quotients  $\frac{3}{2}, \frac{9}{8}, \frac{2}{6}, \frac{1}{3}$  etc. should have a different morphological value from  $\frac{1}{2}, \frac{4}{4}, \frac{8}{4}, \frac{2}{1}$  etc. will remain forever unknown to the intellect if the psyche does not come to its aid, recognizing the first ratio series as modifications of fifth intervals, the second as modifications of the keynote or octave.

I use the term "generator" for the generative element of this selection theorem in order to indicate it as the actual internal form-creating principle (on this, see my *Grundriß*, theorems 20, 21, and 22!).

Regarding this double system—i.e. the fact that sound-images of biological forms are obtained only when one connects two partial-tone coordinate systems at their generator-tones, or at least splits one system into its  $\frac{n}{1}$  and  $\frac{1}{n}$  domains—this becomes clear when we examine the *logarithmic* arrangements of the harmonic tone system, which we shall do next.

If the exponentiated value 8 ( $= 2^3$ ) and the base number 2 are given, then the first step is to find the exponent needed for the base value (2) in order to raise it to the exponentiated value (8). This exponent (3) is called the logarithm of the number 8 in base 2. This is the mathematical definition of "logarithm," summarized thus: The logarithm of  $x$  (the numerus) is that number to the power of which one must raise the base number in order to obtain  $x$  (the

numerus).

This definition becomes clearer in the following series:

2	4	8	16	32
$2^1$	$2^2$	$2^3$	$2^4$	$2^5$

Here, the exponents in the lower series are the base 2 logarithms of the numbers in the upper series. Thus it becomes apparent that the logarithms of an arithmetical series (1 2 3 4 5) and their numeri form a geometric series (2 4 8 16 32).

This relationship in the two series above is characteristic for the relationship between frequency and pitch, or in harmonic language: between material vibration number and psychical tone evaluation (tone-number and tone-value).

If we place the corresponding tone-perceptions or pitches ( $h$ ) beneath the vibration numbers ( $n$ ) of the  $c$ -values perceptible to our ears:

	$c_{//}$	$c_{/}$	$c$	$c'$	$c''$	$c'''$	$c''''$	$c'''''$
$n =$	64	128	256	512	1024	2048	4096	8192
$h =$	0	1	2	3	4	5	6	7
	$2^6$	$2^7$	$2^8$	$2^9$	$2^{10}$	$2^{11}$	$2^{12}$	$2^{13}$

then the lower series of powers shows us that here, we are actually *hearing* the logarithms of base 2, i.e. the first, second, and third octaves, and so on, while the vibration numbers progress in powers of 2, in geometric progression. One can see the prototype of these relationships in a piano: the keys embody the logarithmic element, the part we hear, while the strings represent the precise lengths, or the vibrations of the tones, hence the harp-like shape of the piano, since the vibrations increase in geometric proportion. On this, F. Auerbach writes in his book *Tonkunst und Bildende Kunst* (Jena, 1924, p. 32 ff.), which is worth reading: "Here one must make it clear that the one result is of a quite peculiar and perhaps unexpected nature; after establishing that the first octave reaches from 64 to 128 vibrations, one might have expected the second octave to reach from 128 to 192, and so on. In this respect, however, our results adhere to universal laws also valid in other domains: we do not perceive equal differences to be the same, but rather equal ratio values; it is not a matter of the absolute differences, but of the relative differences, and relatively speaking the numbers 128 and 256 are only as different from one another as the numbers 64 and 128, in each case according to the base number. In mathematics this relationship can be expressed in a very simple and rigid manner, namely with logarithms, those known quantities which in the practical sense have the important function of facilitating calculation. But here the logarithm does not play a practical role, but a fundamental, perception-theoretical one. It allows the law of pitch to

establish, on the basis of the fact that when we take the logarithms of numbers increasing in geometric progression, these logarithms do not increase in geometric progression but in arithmetical progression.”

In the relationship between tone-number and tone-value, i.e. in the core of the tone phenomenon itself, the logarithmic element plays a “fundamental, perception-theoretical” role—and for this very reason, we will apply it for the further development of harmonic sound-images.

This logarithmic element also emerges before us from the biological side, namely as the legitimate expression of organic growth. Brailsford Robertson (see his essay, “On the normal rate of growth of an individual and its chemical significance,” in *Archiv für Entwicklungsmechanik*, vol. 25, 1908) finds this result: “In any given growth cycle, an organism or a specific tissue has its maximal growth in terms of volume or mass per time unit when the cycle is half completed. Every growth cycle follows the formula  $\log \frac{x}{A-x} = K(t - t_1)$ , where  $x$  is the amount of growth (by weight or volume) that is reached at time  $t$ .  $A$  is the total growth during the cycle, where  $K$  is a constant and  $t_1$  is the time at which the growth is half over.”

Logarithmic formulas are found here and there in growth analyses throughout the literature; the base number  $e$  of the natural logarithm system is known as the “number of organic growth.”

I have allowed myself to dwell this long on the logarithmic element because I felt it my duty to draw the friendly reader away from the horror usually felt upon seeing the word and concept “logarithm.” The above examples relationship between tone perception and vibration number is so simple and transparent, as well as being psychically understandable, that this horror will surely fade away to be replaced by an resentment-free attitude. Furthermore, in quoting the two passages above, I wished to show that the following logarithmic representation of the sound-image of plants will be legitimized both on the basis of harmonics and on the basis of organic growth, the latter helping to create the form—not as an external accident, but as fundamental inner factor in both domains.

If we want to draw the partial-tone coordinates as we hear them, i.e. the octaves at equal distances:

$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{4}{1}$	$\frac{8}{1}$
$c_{///}$	$c_{//}$	$c_{/}$	$c$	$c'$	$c''$	$c'''$

then we must illustrate them logarithmically; Table VI in *Hörende Mensch* shows this. In Figure 14, the “completed” partial-tone coordinate diagram is shown in logarithmic arrangement (see *Grundriß*, pp. 100 and 122). As one can see, the tone-numbers are unchanged, but the coordinate grid exhibits logarithmic diminution toward the top and

bottom, which results in the equal distancing of the octaves.

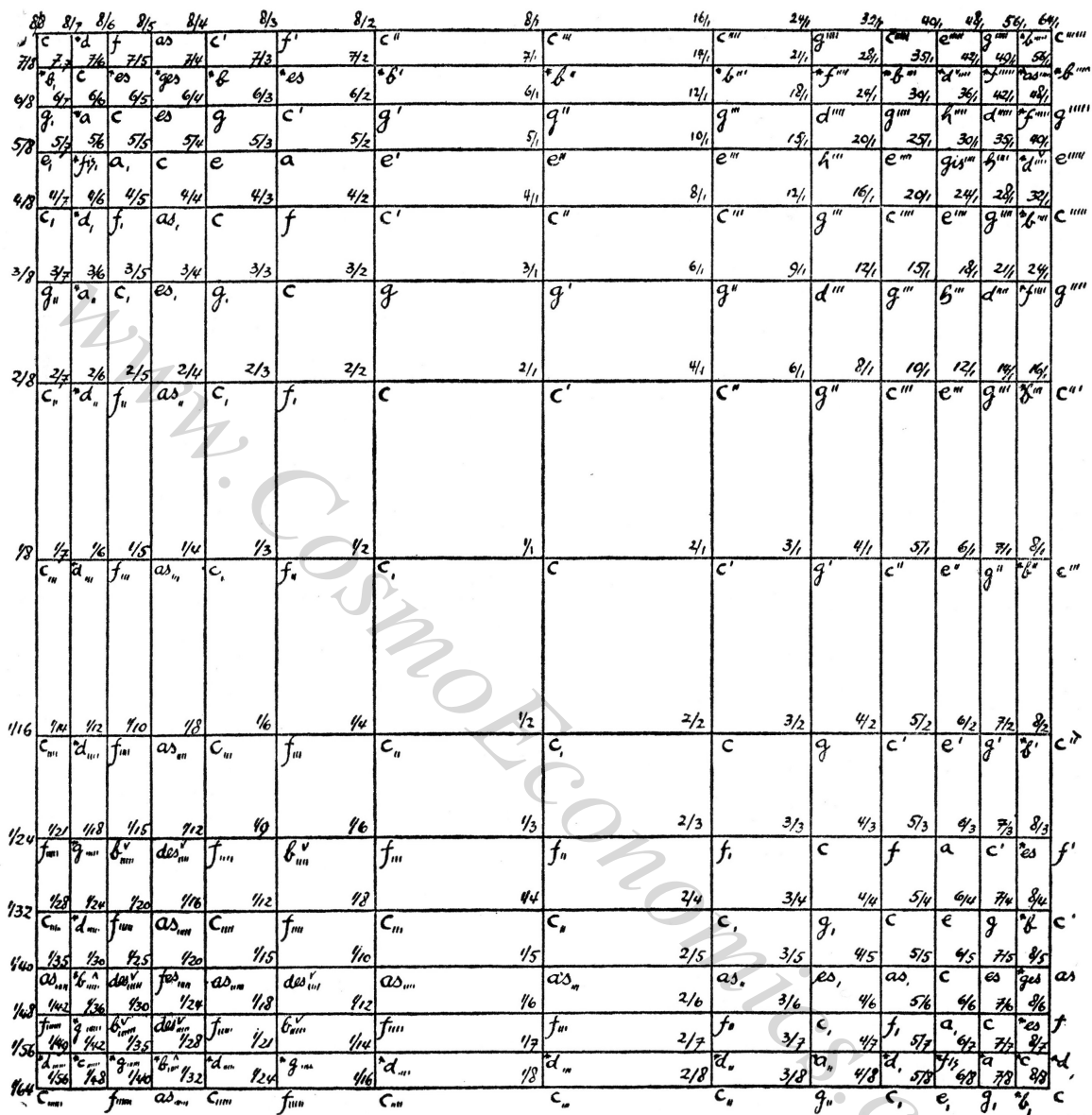


Figure 14

In Figure 15, two quarters of Figure 14 are isolated, with the  $> 1$  values above and the  $< 1$  values below. This diagram has two chief peculiarities: firstly, the perspective diminishing element towards the top and bottom, i.e. directed away from the generator-tone; secondly, all identical tone lines are parallel, i.e. all lines connecting equal tone-values (e.g.  $\frac{8}{4} c' \frac{6}{3} c' \frac{2}{1} c'$  etc.) are parallel to the generator-tone line in the middle (here, for clarity's sake, these parallel lines are drawn only below the  $c$ -values); and thirdly, the "overtone area" above and the "undertone area" below.

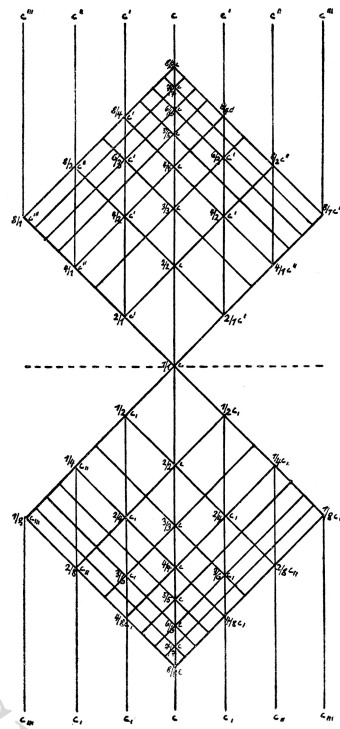


Figure 15

The fact that the *roots* initially have the same morphological structure as the stem, in the prototype of the plant planned by the creative force, is shown by paleobotanic plants such as the roots of lepidophytes, and especially the remarkable *Sigillaria* found in fossils, whose rhizomes still have their core (see *Handwörterbuch der Naturwissenschaften*, 1<sup>st</sup> ed., vol. VII, p. 433).

Setting Figure 15 in analogy to the basic plant form, we find, above all else, an important element of comparison: that of limitation, which takes place via logarithmic diminution and determines a limiting value at the top and the bottom.

However, we come closer to the secret of this basic form when we align the two logarithmic *P*-systems connected at the generator-tone, as shown in Figure 16. This diagram was drawn on ordinary logarithm paper, so that this “aligning” would be visible: the logarithmic division of the paper (doubled division, 1 : 100) begins at the bottom and goes up to the top. The direction of the diagram therefore does not go from the middle  $\frac{1}{1}$  upwards and downwards as in the previous diagram, but instead in only one direction: from the bottom to the top.

For this reason, the  $\frac{1}{1}$  of the above system must be set equal to the  $\frac{10}{10}$  of the lower system; at the center of the diagram, there is a limitation index (here  $\frac{10}{10}$ ), which is identical with 1, the generator-tone of the diagram.

Now, for a better interpretation of the content, we shall allow the diagram to go through a few indexes between 1 and 10 (see Figures 17 and 18).

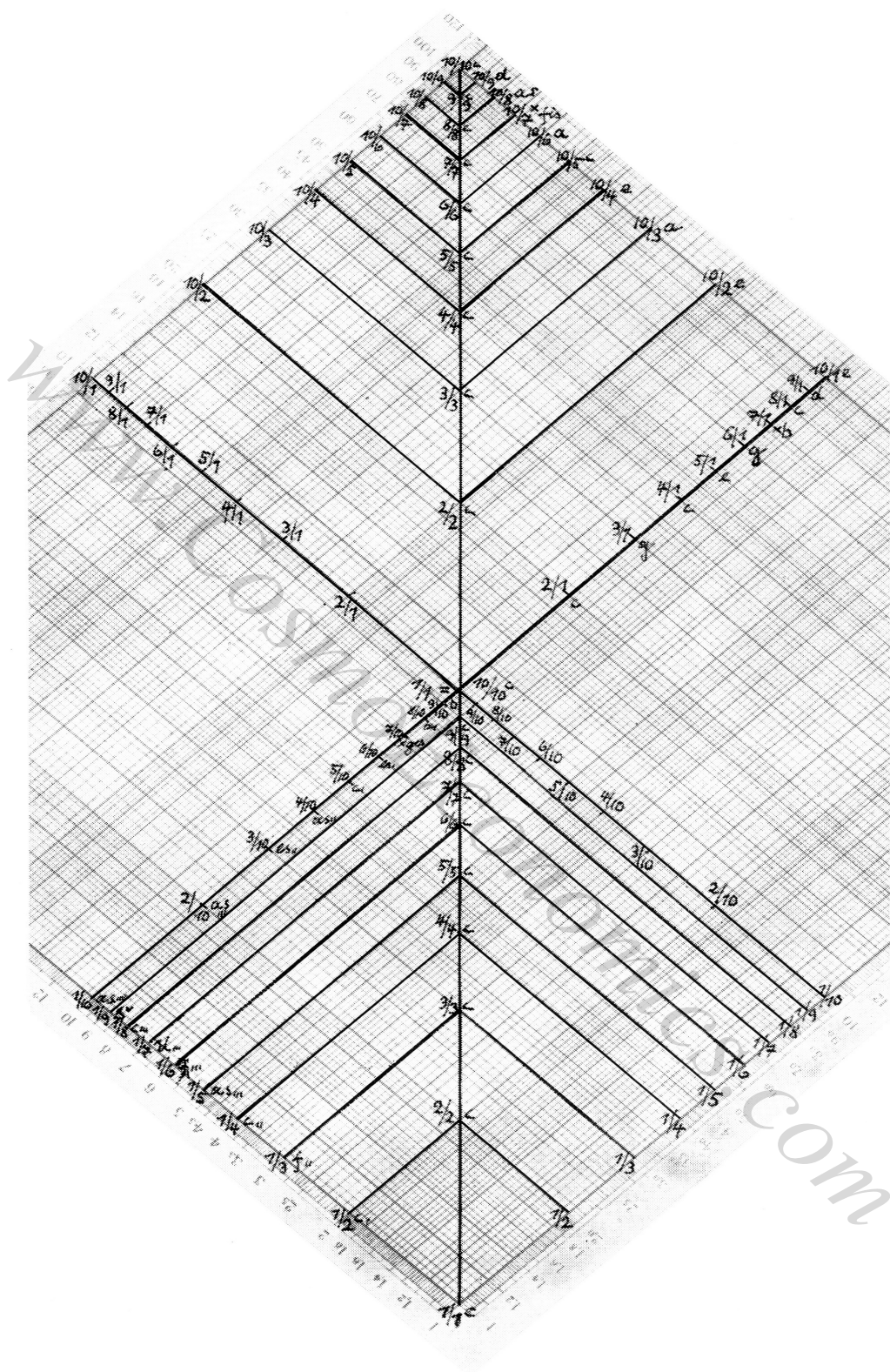


Figure 16

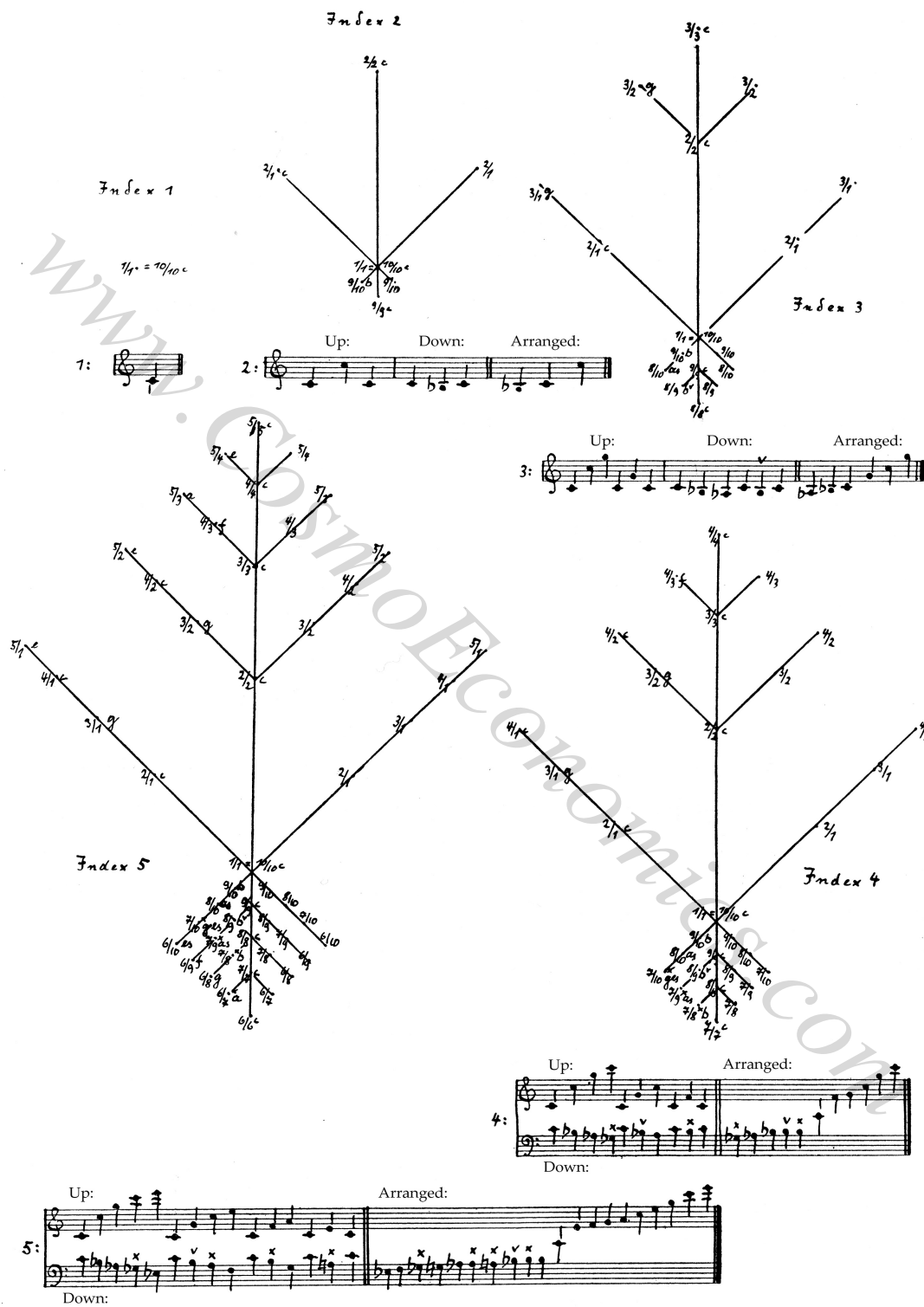


Figure 17



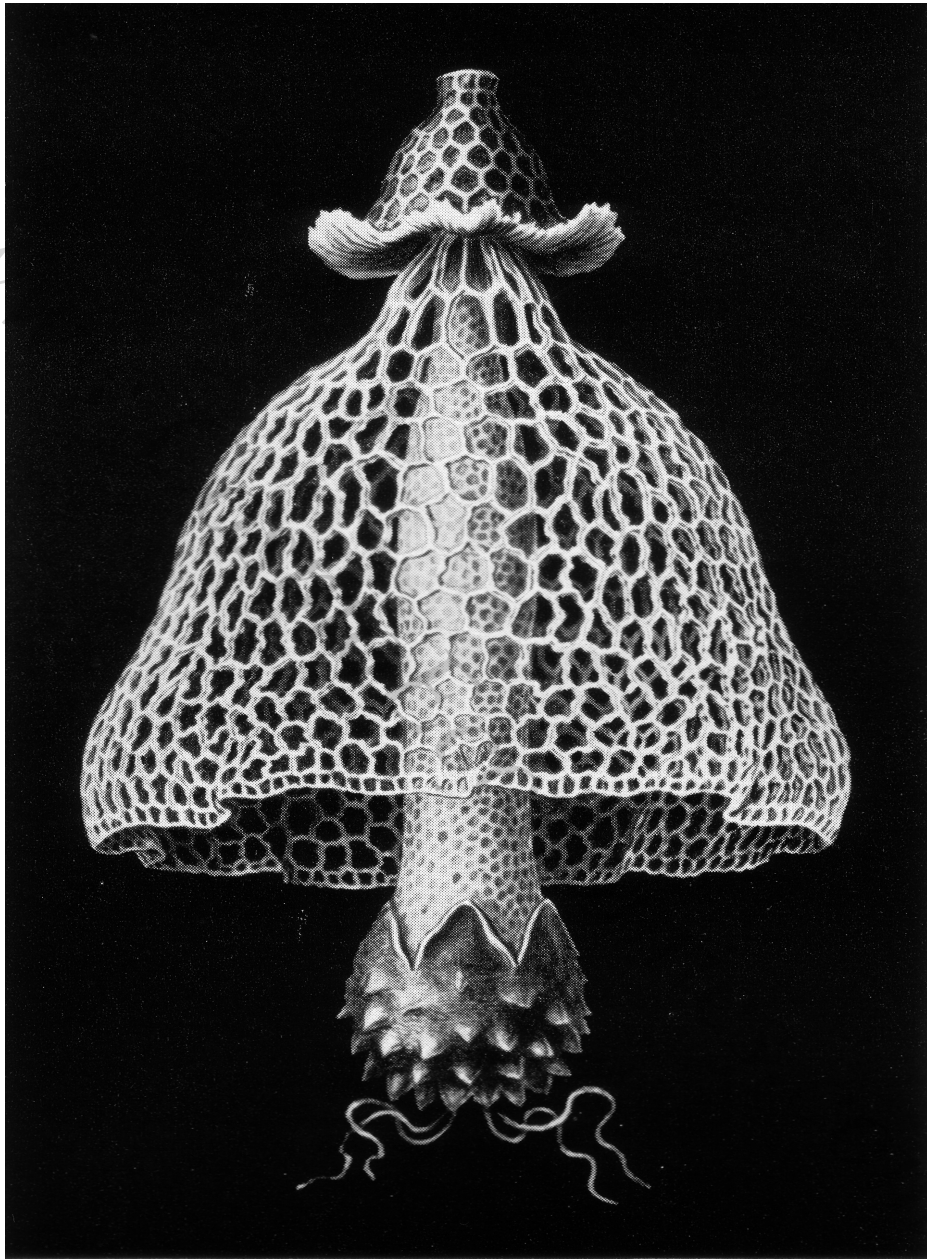


Figure 24

If we now isolate the “most extreme” case in Figure 27 and note the outline ratios of this leaf type, we find the interesting series:

$\frac{6}{1}$	$\frac{6}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{9}{7}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{10}{10}$
$g''$	$g'$	${}^xe b'$	${}^xb b$	$ab$	$f$	${}^xe$	$d$	$d^\vee$	$c$

Its harmonic, i.e. numeric *and* tonal legitimacy is so obvious that here one could speak of a new, previously unknown possibility of arriving by an exact way at an inner, psychical, and also intellectual understanding of the mystery of leaf forms in themselves. Admittedly, this is merely a first beginning, but the leaf forms are there, albeit highly varied, yet not unlimited in their variability; and if a complete system of harmonic sound-images exists, or in modern language, if harmonic group theory exists, then a complete derivation of the leaf model from inner harmonic laws will also be possible.

To give future researchers pointers for these possibilities, I would like to give another harmonic leaf construction which is likewise based on the logarithmic arrangement of the “P” (partial-tone series).

As a basis for this construction, we will use a connection of logarithmic division—or as I say in general elsewhere, “perspective division”—with equidistant division. These two forms of view are, as I endeavored to show in my essay “Die harmonikale Perspektive” (*Abhandlungen*, pp. 41-58), fundamental for every harmonic observation, since they are found *in nuce* in the primal phenomenon of tone-number: the frequency progresses in equal distances (= “equidistant”) while we hear “logarithmically,” or, expressed visually, “perspectively.” (I emphasize “visually” since it is primarily a matter of the “abatement,” the “diminution element,” which is common to perspective and logarithm, although the two obey different mathematical formulas.)

If we now assume that these two elements are also contained in unified form in the growth process, namely so that the impulse of growth in length is logarithmic and the impulse of growth in width is equidistant, then we obtain a leaf form model such as is shown in Figure 28.

Logarithmic paper with vertical logarithmic and horizontal equidistant division is used as a basis. Whereas the logarithmic division always remains the same from top to bottom, the equidistant horizontal division must be established ahead of time for every modification of this leaf type. For example, for the three modifications shown in Figure 28, in *a* (left) I used the section  $\frac{9}{1} - \frac{10}{2}$  or  $\frac{9}{1} - \frac{10}{1}$  (above at the leaf tip) as a measure for the growth in width; in *b* (middle) I used half of this; and in *c* (right)  $\frac{1}{10}$  of this step. If I now draw, from the base  $\frac{1}{1}$  above, the ratios  $\frac{1}{1}$  to  $\frac{10}{1}$ , and go on back from there regularly according to the three aforementioned step widths, to the left and right of the leaf base, then the result is the three figures 28 *a*, *b*, and *c*, whereby it is obvious that I can vary these modifications both in terms of width and in terms of the diminution, up to a limitation value established by nature herself. The leaf *type* always remains the same.

width at  $\frac{1}{2} \times 10 = 5$ , the *four* possibilities drawn in the diagram. First, we go along the generator-tone (leaf) axis  $\frac{1}{1} \frac{2}{2} \frac{3}{3} \dots$  up to  $\frac{10}{10}$ , to the index limit or leaf tip. Then we look from there, down and to the left and right, for the “way” to the ratios  $\frac{5}{1}$ . Since there is no direct possibility here, we must decide on a compromise and take the “two-step” way, from  $\frac{10}{10}$  through  $\frac{9}{8}$ ,  $\frac{8}{6}$ ,  $\frac{7}{4}$ , and  $\frac{6}{2}$ , not by  $\frac{6}{1}$  or  $\frac{5}{1}$  but between these two ratios through an imaginary point that does not “exist.” Of course, we could have chosen the ratio  $\frac{6}{1}$  instead of  $\frac{5}{1}$  as the

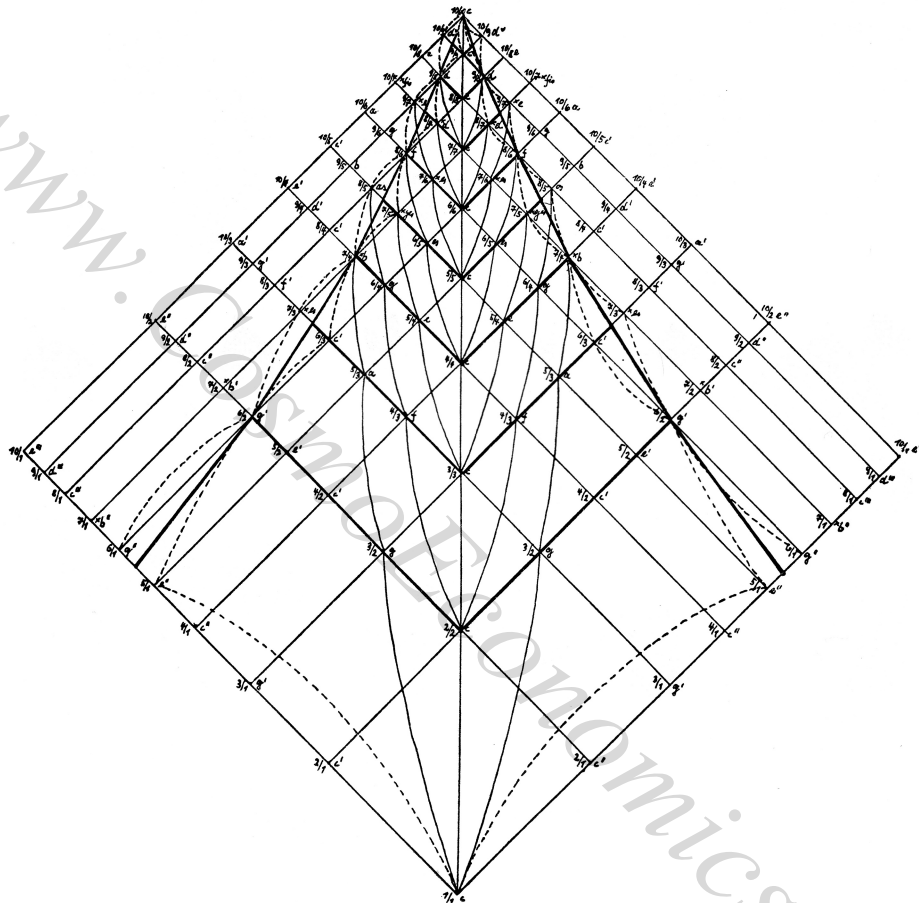


Figure 27

lateral border, but as we will see, it is precisely this that gives us the possibility of (or explanation for) “teeth” or “indentation” on the leaf’s *edge*. Namely, the above “way” from  $\frac{10}{10}$  through  $\frac{9}{8}$ ,  $\frac{8}{6}$ ,  $\frac{7}{4}$ , and  $\frac{6}{2}$  “skips” a leaf vein, as we can see, since here no ratio “meets.” Only when we opt for “recesses” or “bulges” do we find corresponding ratios on the skipped veins. If one now draws these recess or bulge lines not with the ruler but as nature makes them, namely convex or concave, then three possibilities for various leaf forms emerge, which Figure 27 shows in the dotted recess and bulge lines, with the leaf figure drawn in bold. A fourth possibility, whose “leaf formula” will amount to something quite different but likewise precisely findable, is drawn in the middle in thin lines and has a form similar to a single chestnut leaf.

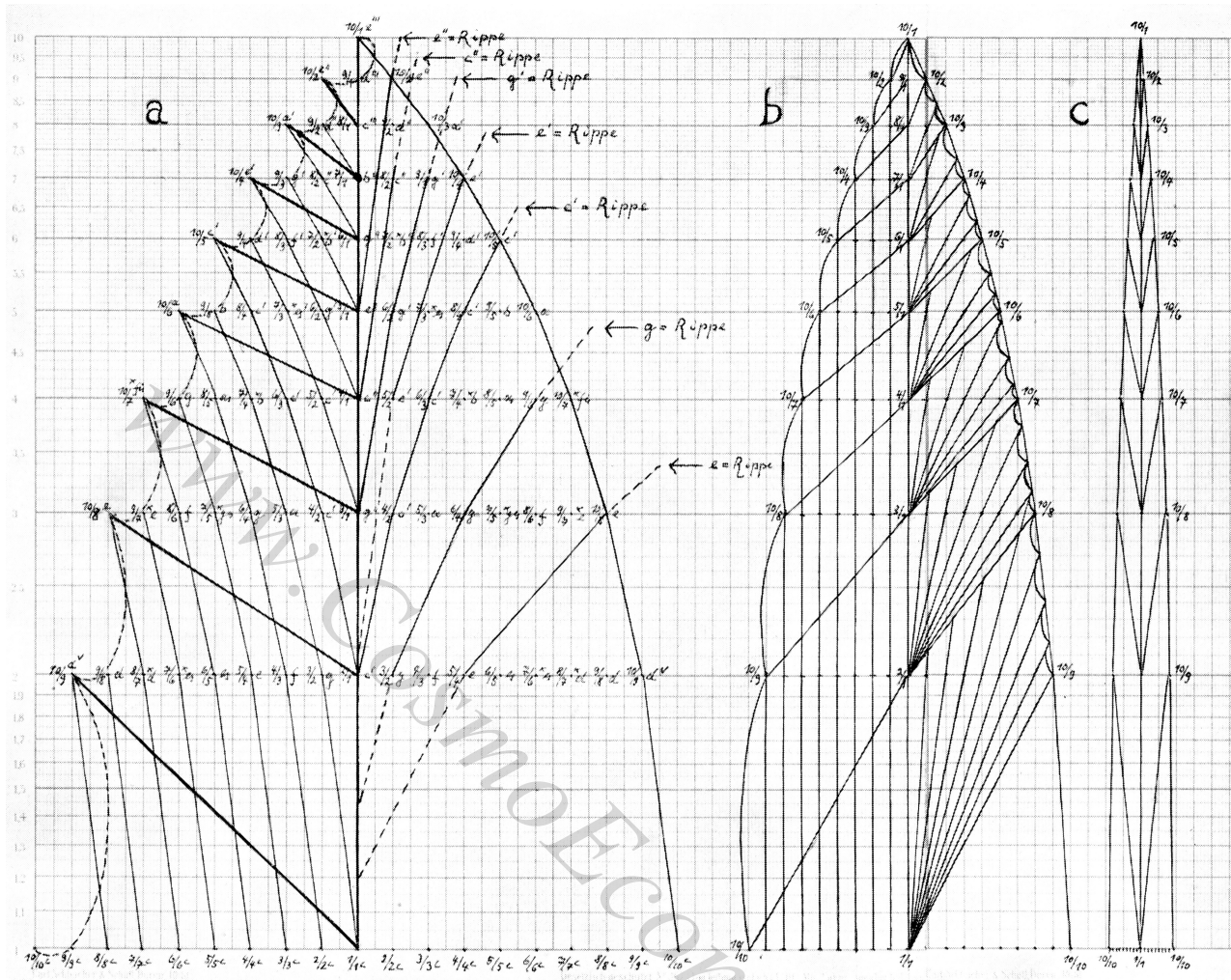


Figure 28

Within this type there are whole groups of very interesting possibilities, especially related to the inner structure of the leaf (veins, venation). For reasons of clarity, I have drawn the ratios completely only in Figure 28 a; in 28 b and c they are at the same places, but pushed closer together.

Since the leaf form is determined by the structure of the veins—"the varied ratios of the veins are the elegant cause of the diverse leaf forms," wrote Goethe in his *Metamorphosis*, §20—we must first examine the various possibilities of this venation or vein arrangement, vein bifurcation.

#### a) "Interval veins"

These are drawn in the left half of Figure 28 a as upward slanting lines, and in the left half of b as downward sloping lines. The "vein forms" of these interval veins in Figures 28 a and b would be the following:

a	b
$^{10}/_1 c'''$	$^{10}/_2 c'' - ^{10}/_1 c'''$
$^9/_1 d'''$	$^{10}/_3 a' - ^9/_1 d'''$
$^{10}/_2 c'' - ^8/_1 c'''$	$^{10}/_4 c' - ^8/_1 c'''$
$^{10}/_3 a' - ^7/_1 b b'''$	$^{10}/_5 e' - ^7/_1 b b'''$
$^{10}/_4 c' - ^6/_1 g''$	$^{10}/_6 a - ^6/_1 g''$
$^{10}/_5 e' - ^5/_1 e''$	$^{10}/_7 f\sharp - ^5/_1 e''$
$^{10}/_6 a - ^4/_1 c''$	$^{10}/_8 c - ^4/_1 c''$
$^{10}/_7 f\sharp - ^3/_1 g'$	$^{10}/_9 d\sim - ^3/_1 g'$
$^{10}/_8 c - ^2/_1 c'$	$^{10}/_{10} c - ^2/_1 c'$
$^{10}/_9 d\sim - ^1/_1 c$	$^1/_1 c$
$^{10}/_{10} c$	

As one can see, and as the figures show, the intervals are the same in Figure 28 *a* and *b*, only connected to one another with different “vectors.” In these modifications one must assume that the form-determining ratio beginning seeks the progression according to the “limiting value” of the next step up or down starting from  $^1/_1$  or  $^{10}/_{10}$ , which produces the various vectorial directions. Surprising, although naturally determined by the logarithmic partial-tone coordinates, is the vein image (i.e. various vein angles), at first steeply climbing upwards (descending in *b*), then inclining somewhat toward the horizontal and moving again toward the middle axis near the tip. I had observed this in a large number of leaf forms, and here it is explained as a law present *a priori* in the inner structure of this leaf model. These “interval veins,” the modifications of Figures 28 *a* and *b*, could be drawn something like this (Figure 29):

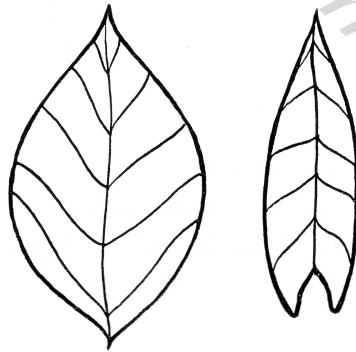


Figure 29

#### b) “Hierarchal veins.”

These are drawn upwards in fan-shaped form on the left half of Figure 28 *a* from the generator-tone base  $^1/_1 c \ ^2/_2 c \dots ^{10}/_{10} c$ . The vein starting from the base ratio  $^2/_2 c$ , for example, goes through  $^4/_3 f - ^6/_4 g - ^8/_5 a b$  to  $^{10}/_6 a$ ; the vein from the base  $^3/_3 c$  goes through  $^5/_4 e - ^7/_5 g b$  to

(perspectively)—an assumption that is not arbitrary, being permitted in harmonic thinking since in the system of the partial-tone series the overtone and undertone series overlap. We now obtain the bifurcation diagram in Figure 34.

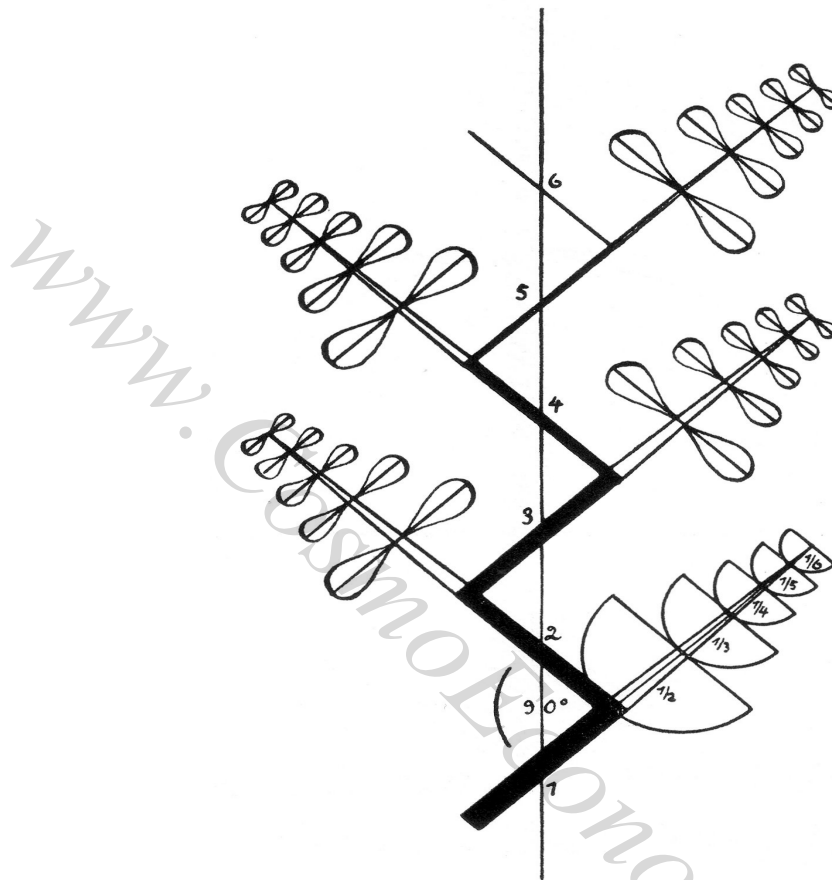


Figure 34

The index of the diagram is limited to 6, the measurement sets the lowest part of the stem at 1. The stem, growing upward in  $90^\circ$  zigzags, therefore proceeds in minor-tonal measures,  $1\ c\ 2\ c_1\ 3\ f_{II}\ 4\ c_{II}\ 5\ a\ b_{III}\ 6\ f_{III}$ ; the leaf angle nodes according to major-tonal measures,  $1\ c\ 1/2\ c'\ 1/3\ g'\ 1/4\ c''\ 1/5\ e''\ 1/6\ g''$ . As one can see, the stem thickness, which is set at 5 ( $6 - 1$ ) units in the lowest internode, diminishes regularly ( $4 = 6 - 2$ ;  $3 = 6 - 3$ ;  $2 = 6 - 4$ ;  $1 = 6 - 5$ ) upwards until it disappears into a “line” with no more width; the same element is indicated in the leaf stems. The first, lowest leaf on the right shows the law-based construction; for the others, real leaves are drawn, in order to show the leaf or bifurcation model corresponding to this diagram. Obviously, other types of leaves could be substituted. The characteristic and decisive element here is the bifurcation *model*.

We now come to another model, in which the ascension *angle* of bifurcation is legitimately included. As a harmonic prototype for this and the following models, we use *only* the division

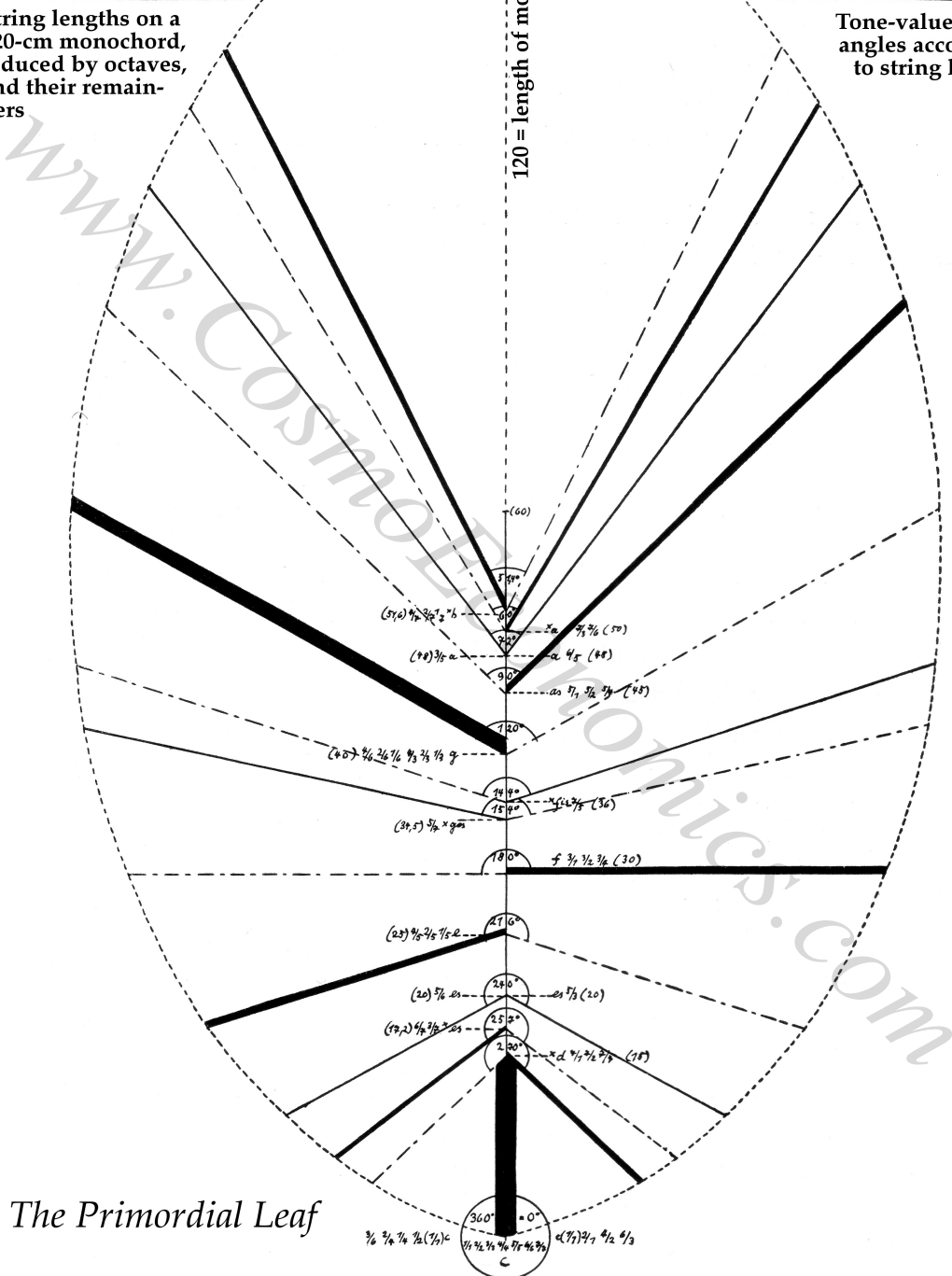
120	120	90	120	95	90	105
60	60	30	60	45	30	15
120	120	90	120	95	90	105
60	60	30	60	45	30	15
80	80	120	80	100	120	50
40	40	60	40	20	60	70
120	120	90	120	95	90	105
60	60	30	60	45	30	15
96	96	92	96	120	92	84
24	24	48	24	60	48	36
80	80	120	80	100	120	50
40	40	60	40	20	60	70
68.8	68.8	102.8	68.8	83.5	102.8	57
51.6	51.6	77.2	51.6	34.5	77.2	60

### String lengths on a 120-cm monochord, reduced by octaves, and their remainders

**P**  
**E**<sub>7</sub>

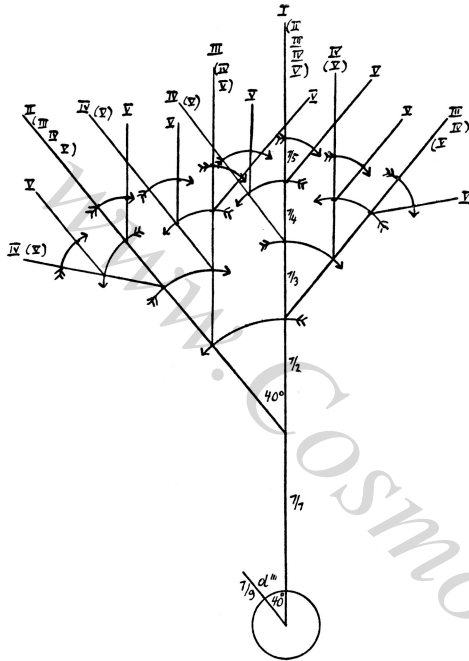
$\frac{1}{4}c$ 0°	$\frac{3}{4}c,$ $\frac{1}{8}0°$	$\frac{1}{2}f$ $\frac{1}{8}0°$	$\frac{1}{4}a,$ $\frac{1}{8}0°$	$\frac{1}{2}a,$ $\frac{1}{8}0°$	$\frac{1}{4}f,$ $\frac{1}{8}0°$	$\frac{1}{2}f,$ $\frac{1}{8}0°$	$\frac{1}{4}a,$ $\frac{1}{8}0°$
$\frac{1}{2}c,$ 0°	$\frac{1}{4}c$ $\frac{1}{8}0°$	$\frac{1}{4}c,$ 0°	$\frac{1}{2}c,$ $\frac{1}{8}0°$	$\frac{1}{4}f,$ $\frac{1}{8}0°$	$\frac{1}{2}f,$ $\frac{1}{8}0°$	$\frac{1}{4}a,$ $\frac{1}{8}0°$	$\frac{1}{2}a,$ $\frac{1}{8}0°$
$\frac{3}{4}f,$ $\frac{1}{2}0°$	$\frac{1}{2}f$ $\frac{1}{2}0°$	$\frac{1}{4}c$ $\frac{1}{2}0°$	$\frac{1}{4}f,$ $\frac{1}{2}0°$	$\frac{1}{2}f,$ $\frac{1}{2}0°$	$\frac{1}{4}a,$ $\frac{1}{2}0°$	$\frac{1}{2}a,$ $\frac{1}{2}0°$	$\frac{1}{4}f,$ $\frac{1}{2}0°$
$\frac{1}{4}c$ 0°	$\frac{1}{4}c,$ 0°	$\frac{1}{2}f$ $\frac{1}{8}0°$	$\frac{1}{4}a,$ $\frac{1}{8}0°$	$\frac{1}{2}a,$ $\frac{1}{8}0°$	$\frac{1}{4}f,$ $\frac{1}{8}0°$	$\frac{1}{2}f,$ $\frac{1}{8}0°$	$\frac{1}{4}a,$ $\frac{1}{8}0°$
$\frac{1}{2}c$ $\frac{1}{2}0°$	$\frac{1}{4}c$ $\frac{1}{2}0°$	$\frac{1}{2}c$ $\frac{1}{2}0°$	$\frac{1}{4}a$ $\frac{1}{2}0°$	$\frac{1}{2}a$ $\frac{1}{2}0°$	$\frac{1}{4}f$ $\frac{1}{2}0°$	$\frac{1}{2}f$ $\frac{1}{2}0°$	$\frac{1}{4}a$ $\frac{1}{2}0°$
$\frac{3}{4}f$ $\frac{1}{2}0°$	$\frac{1}{2}f$ $\frac{1}{2}0°$	$\frac{1}{4}c$ $\frac{1}{2}0°$	$\frac{1}{4}f$ $\frac{1}{2}0°$	$\frac{1}{2}f$ $\frac{1}{2}0°$	$\frac{1}{4}a$ $\frac{1}{2}0°$	$\frac{1}{2}a$ $\frac{1}{2}0°$	$\frac{1}{4}f$ $\frac{1}{2}0°$
$\frac{1}{4}c$ 60°	$\frac{1}{4}c$ $\frac{1}{2}0°$	$\frac{1}{2}f$ $\frac{1}{2}0°$	$\frac{1}{4}a$ $\frac{1}{2}0°$	$\frac{1}{2}a$ $\frac{1}{2}0°$	$\frac{1}{4}f$ $\frac{1}{2}0°$	$\frac{1}{2}f$ $\frac{1}{2}0°$	$\frac{1}{4}a$ $\frac{1}{2}0°$

### Tone-values and angles according to string length



**Figure 50**

"4. The specific stylistics (behavior) of the system are determined by the 'conversion rule' (for dicotyledons), which dictates that if we convert from an axis of the 2<sup>nd</sup> order, the following 1<sup>st</sup> axis of the 3<sup>rd</sup> order will be outside the conversion angle, i.e. outside the angle formed by the first two axes. This applies throughout the whole system.

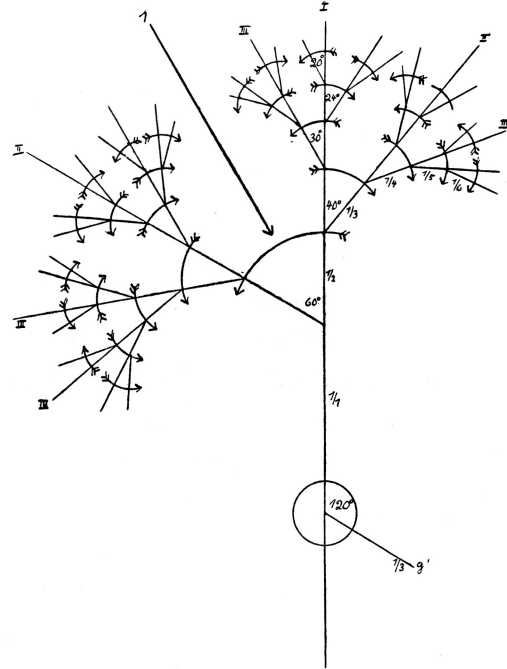


**Figure 53**

"Leaf quotient"  $^{10}/_5 = 2$

Ascension angle  $40^\circ$  constant, but turning right-left.

Segments according to the string lengths  $^{1}/_1 \ ^{1}/_2 \ ^{1}/_3 \ ^{1}/_4 \ ^{1}/_5$ .



**Figure 54**

"Leaf quotient"  $^{21}/_{10} = 2.1$

Ascension angle  $120^\circ$  abating according to the order  $^{1}/_1 \ ^{1}/_2$

$^{1}/_3 \ ^{1}/_4 \ ^{1}/_5 \ ^{1}/_6$ . Arrangement of the right-left rule (arrows) according to Heidenhain's "conversion rule."

"5. The aforementioned asymmetry of the whole and of the individual subordinate bifurcations systems can be illustrated with the designation of numerical 'leaf quotients,' namely by counting the free leaf tips on both sides of each axis in question (1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> order etc.) and dividing the larger number on one side by the smaller number on the other. For example (see Figure 52), on the right side of the 1<sup>st</sup> order axis we obtain 21 free ends, on the left 10.  $^{21}/_{10} = 2.1$ . This quotient, as one can confirm by counting, is always approximately 2 in the case of the geometrical scheme, both in the system as whole and in the subordinate systems of the subsequent axes.

"There are cases in which the theory of the quotients is in agreement with actual algebraically precise calculations (as with the fern *Adiantum*).

"6. The asymmetry of the bifurcation system, expressed in the leaf quotients, is a consequence of the right-left rule and the size of the side twigs, diminishing toward the tips



$$\begin{array}{rcl}
 1 : \frac{8}{5} ab & = & 360 : x \\
 \frac{8}{5} \times 360 & = & 576^\circ \\
 - 360^\circ & & \\
 \hline
 & = & 216^\circ \quad (ab)
 \end{array}$$

In order to find the distances from the circle's center (or from the circle with radius 1) that determine the spiral points, the fractions are converted into decimals:

$$\begin{array}{rclcl}
 4 & : & 3 & = & 1.33 \quad \dots \quad (f) \\
 8 & : & 5 & = & 1.60 \quad (ab)
 \end{array}$$

and these distances are marked on the  $f$  or  $ab$  radius.

In this undertone division, there appear the circle arcs  $f$   $120^\circ (= \frac{1}{3})$ ,  $ab$   $216^\circ (= 3 \times \frac{1}{5})$ ,  $^x d$   $51.4^\circ (= \frac{1}{7})$ , and so on, dividing the circle into three, five, and seven.

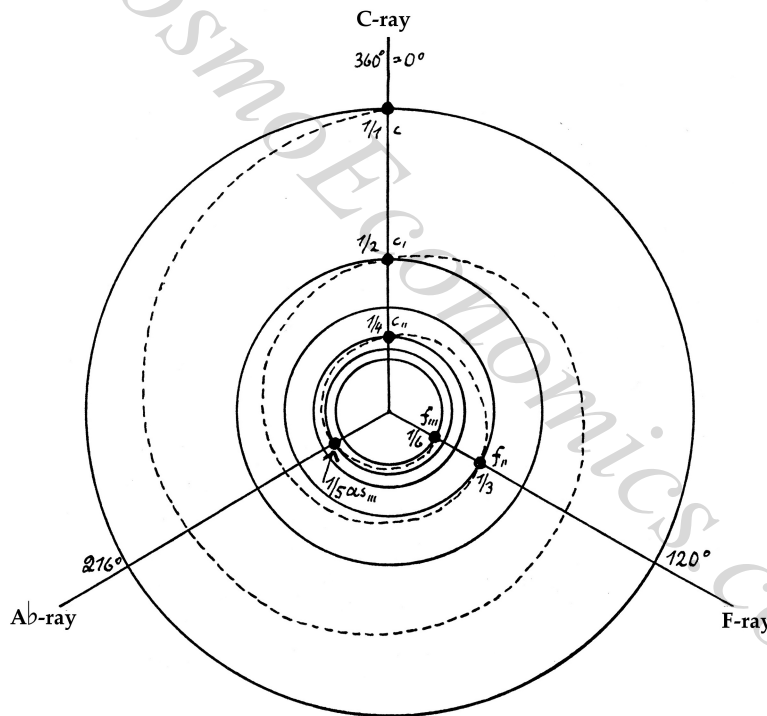


Figure 77

Here, with the circular illustration in frequencies, we see emerging the same element, decisive for the harmonic nature of leaf positioning, as we saw in the above simple divisions of the circle periphery as a string: certain regular figures, indicating the main numbers of the most important leaf positions ( $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$ ,  $\frac{3}{8}$ ,  $\frac{5}{13}$ ) either in their actual numbers or in their intervals (octaves, fifths/fourths, and thirds). The connecting element in both illustrations is,

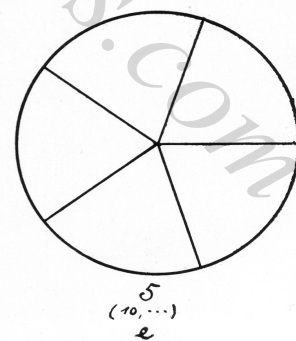
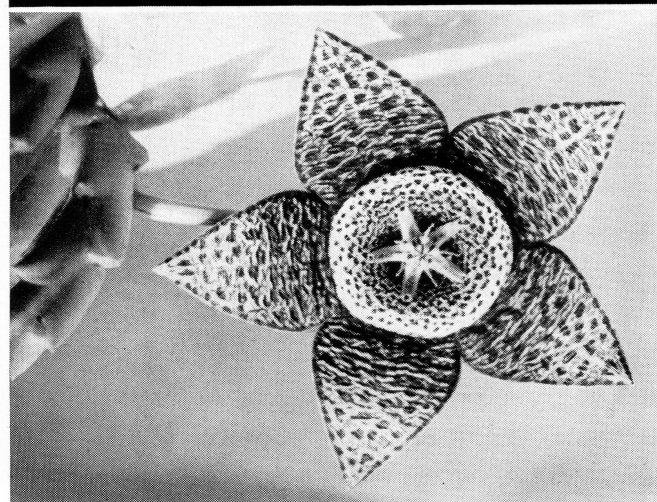
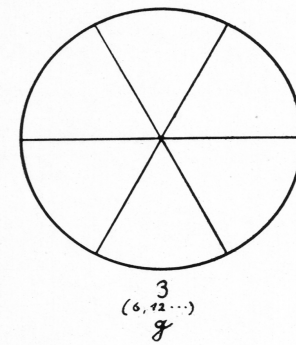
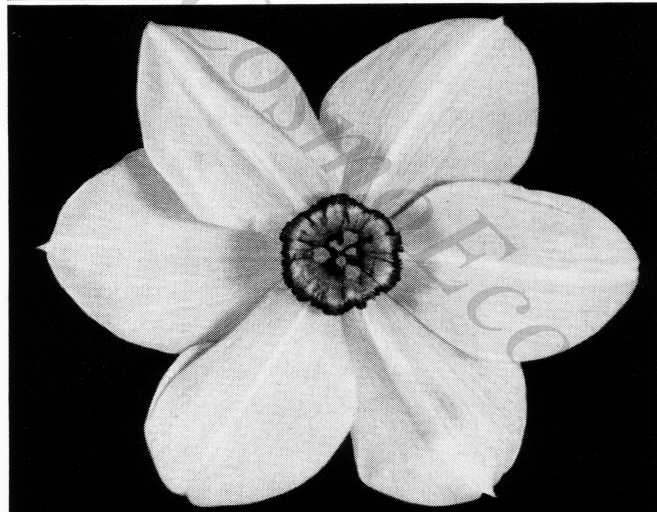
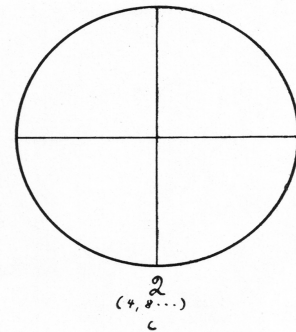
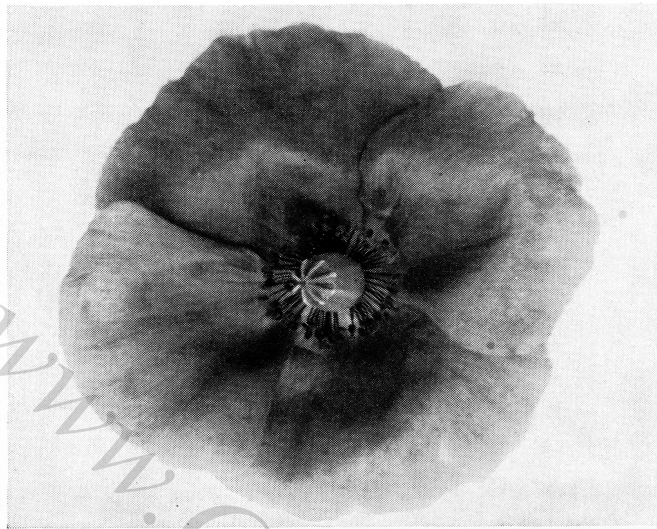


Figure 79

The second diagram (Figure 85) shows the logarithmic analog to Figure 8, and the third (Figure 86) shows the logarithmic analog to Figure 9. Here as well, only the index lines and axes are drawn in to show the outline. These last two diagrams are reciprocal, as can be confirmed by looking at Figures 8 and 9; in the logarithmic diagrams, the form of the outline does not change, but the ratio structure of the axes does.

In Figure 87, a closed triadic arrangement (3, 6) of two different possible arrangements is selected and logarithmically transformed.

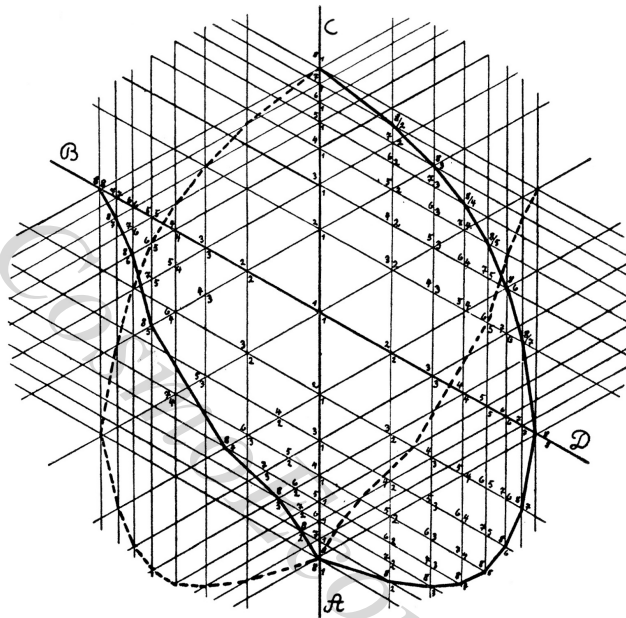


Figure 87

Only index 8 is drawn as an outline, and one can see how finely the rhythm and line differentiation emerges here.

The last four diagrams can be transformed from vertical calyx (bud, flower) forms into horizontal flower plans by drawing each form four or six times.

As further possible harmonic flower constructions, I will give the “flower of the partial-tone parabolas,” which represent the constitutive element of the overtone and undertone series within the  $\frac{1}{4}$  partial-tone plane (cf. *Hörende Mensch* Plate I), and are connected fourfold in Figure 88. The flower-like forms in Figure 89, isolated from the central part of Figure 88, show the morphological fruitfulness of this prototype. Its inner psychical interpretation identifies it as the chord within the flower’s form, whereby the parabolas (Figure 90), which intersect each other reciprocally and always represent the major and minor patterns of the overtone and undertone series respectively, appear as a morphological transformation of the harmonic element itself.

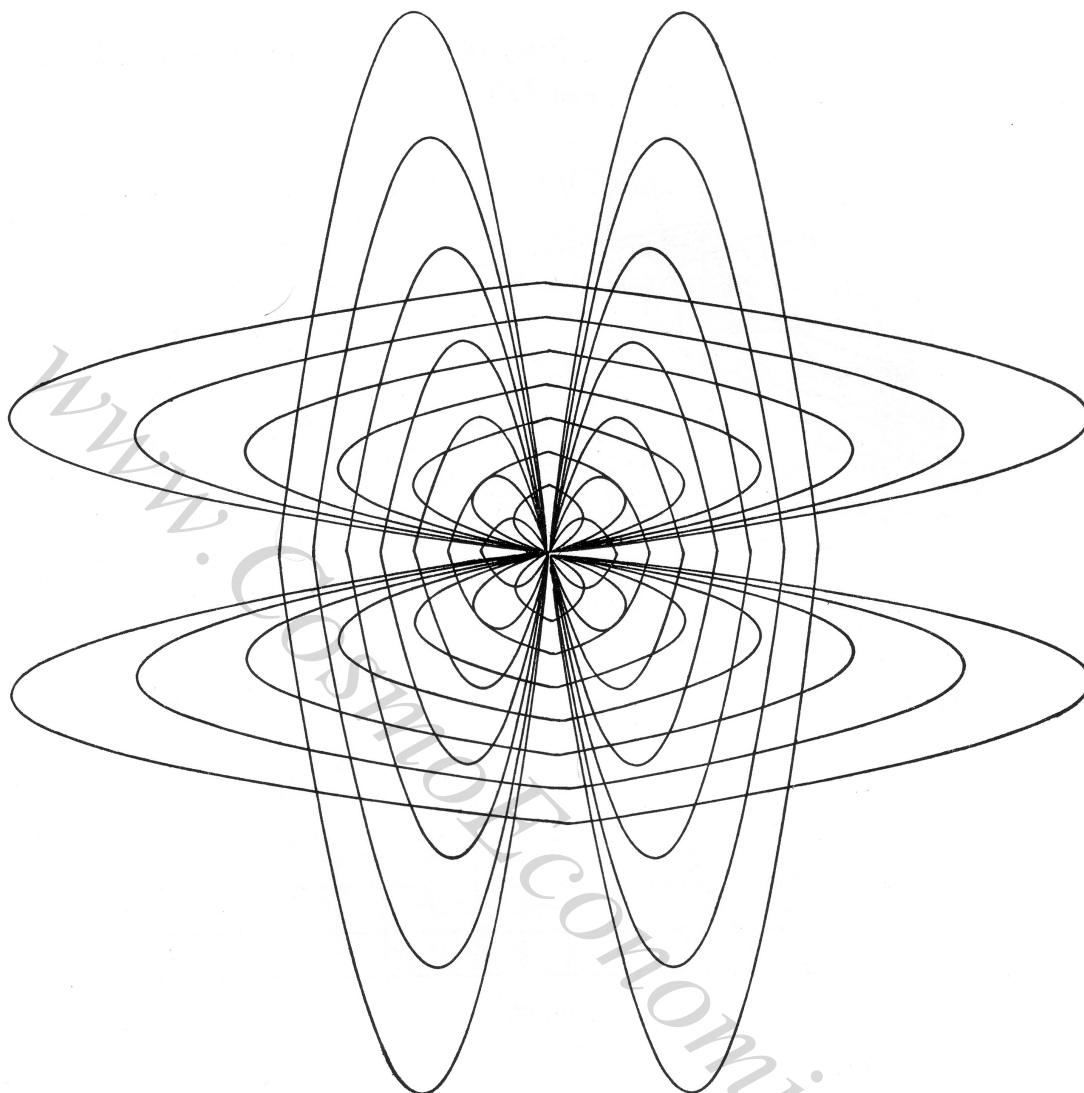


Figure 88

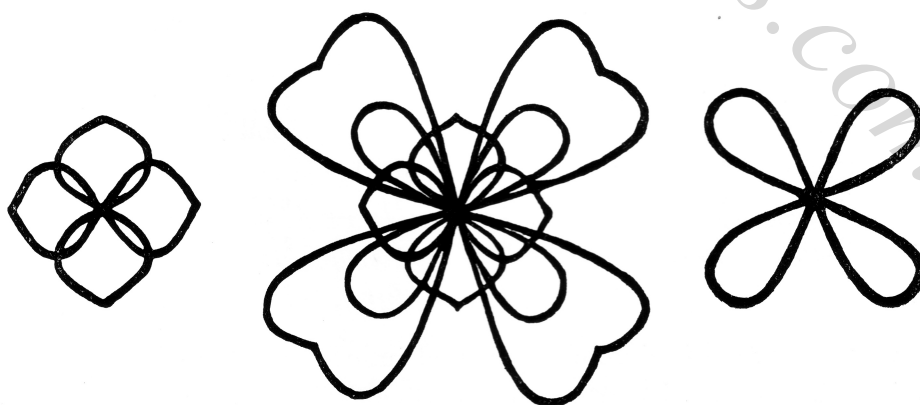


Figure 89

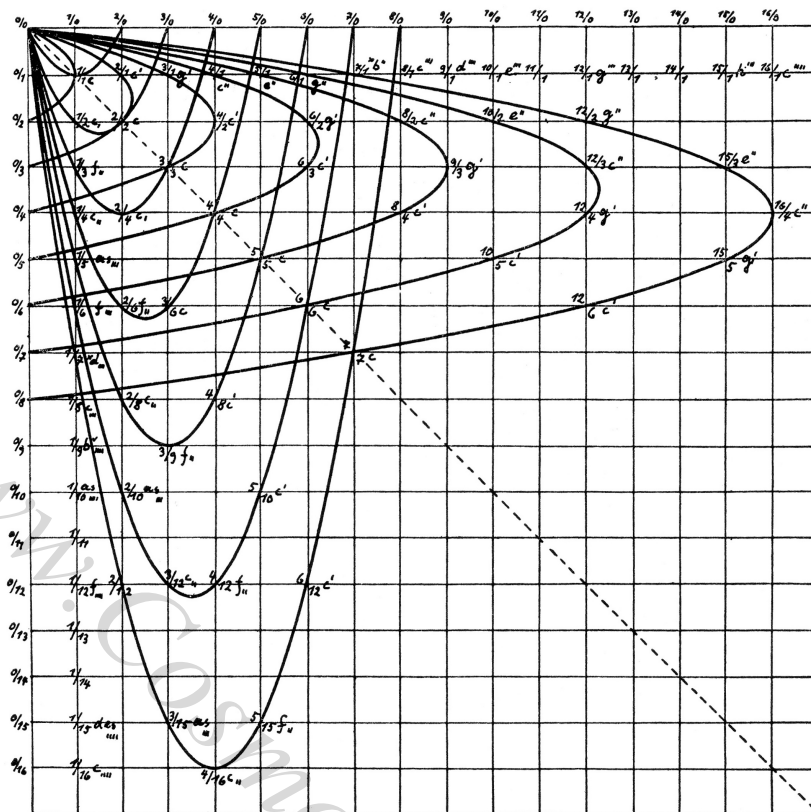


Figure 90

$(^0/2)$	$1/1$	$0/0$	Major			
$(^0/3)$	$2/2$	$2/1$	$(^0/0)$			
$(^0/4)$	$3/3$	$4/2$	$3/1$	$(^0/0)$		
$(^0/5)$	$4/4$	$6/3$	$6/2$	$4/1$	$(^0/0)$	
$(^0/6)$	$5/5$	$8/4$	$9/3$	$8/2$	$5/1$	$(^0/0)$
	$c$	$c'$	$g'$	$c''$	$e''$	etc.
$(^2/0)$	$1/1$	$(^0/0)$	Minor			
$(^3/0)$	$2/2$	$1/2$	$(^0/0)$			
$(^4/0)$	$3/3$	$2/4$	$1/3$	$(^0/0)$		
$(^5/0)$	$4/4$	$3/6$	$2/6$	$1/4$	$(^0/0)$	
$(^6/0)$	$5/5$	$4/8$	$3/9$	$2/8$	$1/5$	$(^0/0)$
	$c$	$c'$	$f''$	$c''$	$a b'''$	